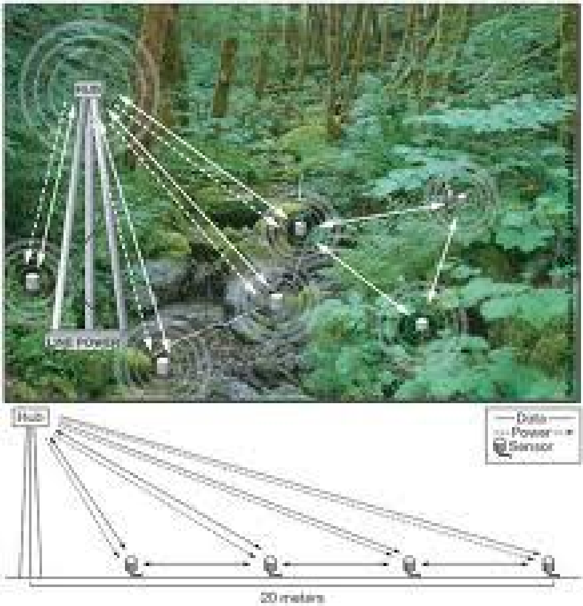
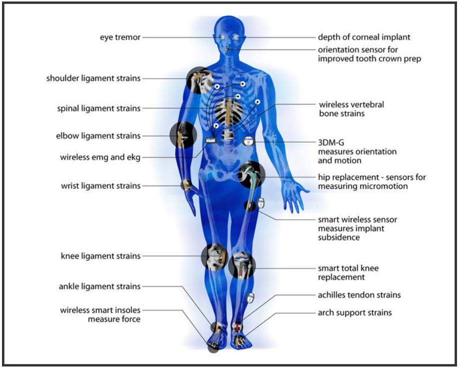
Homologies et réseaux de capteurs

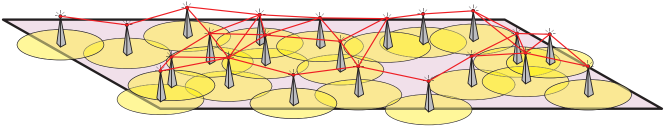
L. Decreusefond

MAT4NET


Hub
Data
Power
Sensor
20 meters


eye tremor
depth of corneal implant
orientation sensor for
improved tooth crown prep
shoulder ligament strains
spinal ligament strains
wireless vertebral
bone strains
elbow ligament strains
3DM-G
measures orientation
and motion
wireless emg and ekg
wrist ligament strains
hip replacement - sensors for
measuring micromotion
smart wireless sensor
measures implant
subsidence
knee ligament strains
smart total knee
replacement
ankle ligament strains
achilles tendon strains
arch support strains
wireless smart insoles
measure force

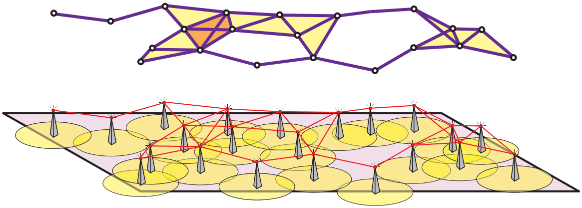
Couverture



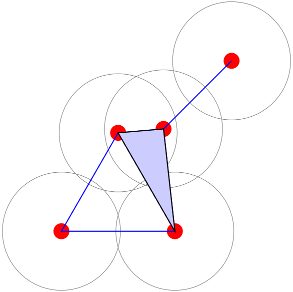
Complexe simplicial

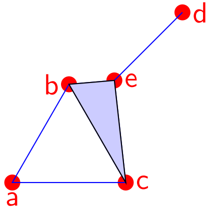
- Généralise la notion de graphes
- Constitué d'arêtes, de triangles, de tétrahèdres, ...

Exemple



Complexe de Cech





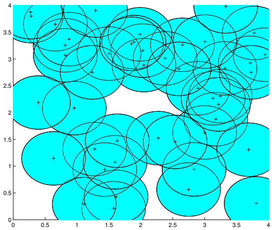
Sommets : $\{$ a, b, c, d, e $\} = \mathcal{C}_0$

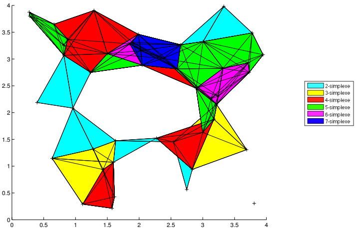
Arêtes : $\{$ab, bc, ca, be, ec, ed $\} = \mathcal{C}_1$

Triangles : $\{$bec$\} = \mathcal{C}_2$

Tétrahèdre : $\emptyset = \mathcal{C}_3$

Exemple plus compliqué





Opérateur de bord

Définition

$$\begin{aligned} \partial_k \ : \ C_k &\longrightarrow C_{k-1} \\ [v_0, \cdots, v_k] &\longmapsto \sum_{j=0}^{k} (-1)^j [v_0, \cdots, \hat{v}_j, \cdots] \end{aligned}$$

Exemple

$\partial(bec) = ec - bc + be$
$\partial^2(bec) = c - e - (c - b) + e - b = 0$

Théorème

$$\partial_k \partial_{k+1} = 0$$

Conséquence

$$\mathrm{Im}\ \partial_{k+1} \subset \ker \partial_k$$

Définition

$$\beta_k = \dim \ker \partial_k - \mathrm{range}\ \partial_{k+1}$$

Interprétation

- β_0 : nb de composantes connexes
- β_1 : nb de trous
- β_2 : nb de « vides »
- ...

Exemple

Rappel :

$$\mathcal{C}_0 = \{a, b, c, d, e\},\ \mathcal{C}_1 = \{ab, bc, ca, be, ec, ed\}$$

$$\partial_0 \equiv 0,\ \partial_1 = \begin{pmatrix} -1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

Nb de composantes connexes

dim ker $\partial_0 = 5,$ range $\partial_1 = 4$ donc $\beta_0 = 1$

Nombre de trous

Rappel :

$$\mathcal{C}_1 = \{ab, bc, ca, be, ec, ed\}, \; \mathcal{C}_2 = \{bec\}$$

$$\partial_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

dim $\ker \partial_1 = 2$, range $\partial_2 = 1$ donc $\beta_1 = 1$

Caractéristique d'Euler

Définition

$$\chi = \sum_{j=0}^{d} (-1)^j \beta_j = \sum_{j=0}^{\infty} (-1)^j \left|\mathcal{C}_k\right|$$

Inégalité de Morse

$$-\left|\mathcal{C}_{k-1}\right| + \left|\mathcal{C}_k\right| - \left|\mathcal{C}_{k+1}\right| \leq \beta_k \leq \left|\mathcal{C}_k\right|$$

- Algorithme centralisé
- Nécessite de connaître les positions exactes

Complexe de Rips

$$[x_0, \cdots, , x_k] \in \mathcal{R}_k(\epsilon) \Longleftrightarrow |x_i - x_j| \leq \epsilon$$

Propriétés

- Si distance $= l^{\infty}$, $\mathcal{C}_k(\epsilon) = \mathcal{R}_k(\epsilon)$
- Pour la distance euclidienne

$$\mathcal{R}_k\left(\epsilon\sqrt{\frac{d+1}{2d}}\right) \subset \mathcal{C}_k(\epsilon) \subset \mathcal{R}_k(2\epsilon)$$

Quelques résultats (D-Ferraz-Randriam-Vergne)

n points, uniformément répartis sur un d-tore d'arête a

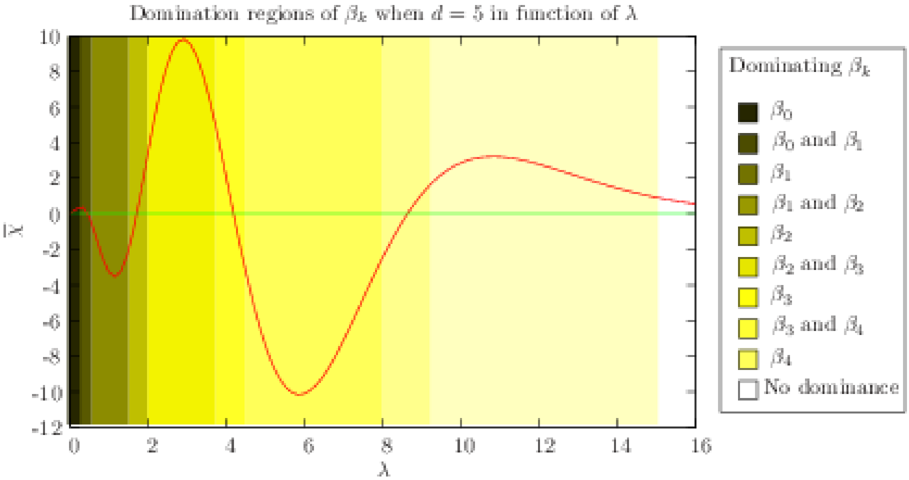
k simplexes

$$\mathbf{E}[|\mathcal{C}_k(n)|] = \binom{n}{k+1}(k+1)^d \left(\frac{2\epsilon}{a}\right)^{dk}$$

Caractéristique d'Euler

$$\mathbf{E}[\chi(n)] = \sum_{k=0}^{n} \binom{n}{k+1}(-1)^k (k+1)^d \left(\frac{2\epsilon}{a}\right)^{dk}$$

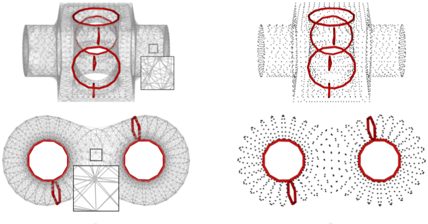
Dimension 5



Domination regions of β_k when $d = 5$ in function of λ
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
$\bar{\chi}$
0
2
4
6
8
10
12
14
16
λ
Dominating β_k
β_0
β_0 and β_1
β_1
β_1 and β_2
β_2
β_2 and β_3
β_3
β_3 and β_4
β_4
No dominance

Mise en œuvre

- Calculs algébriques classiques
- Base « minimale » des e.v. quotients donne les bords des trous



Autre application (D.- Martins- Vergne)

Green networking

Eteindre des capteurs en maintenant la couverture

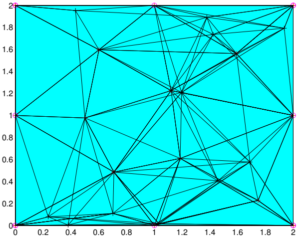
Hauteur d'une arête

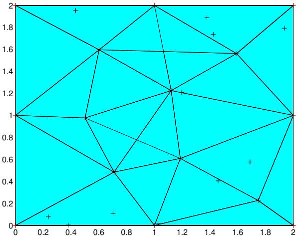
Ordre du plus grand simplexe auquel elle appartient

Indice d'un sommet

Minimum des hauteurs des arêtes adjacentes

Exemple





Complexité (D.-Vergne)

Régime sous-critique

Si

$$\frac{k^{\frac{1+\eta-d}{k-1}}}{n^{\frac{k}{k-1}}} < \theta := \left(\frac{\epsilon_n}{a}\right)^d < \frac{k^{-\frac{1+\eta+d}{k}}}{n^{\frac{k+1}{k}}}$$

alors la hauteur tend vers k quand n tend vers l'infini.

Régime critique

Si $n\theta_n \to 1$ alors

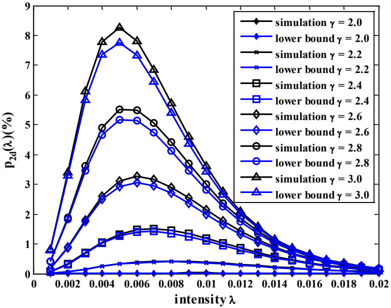
$$(\ln n)^{1-\eta} < \text{hauteur} < \ln n, \quad \forall \eta > 0.$$

Régime sur-critique

Si $n\theta_n \to \infty$ alors hauteur$\sim n\theta_n$.

Rips-Cech (D-Feng-Martins

- Norme euclidienne
- Rayon de couverture R_S
- Rayon de communication R_C



L. Decreusefond, E. Ferraz, H. Randriambololona, and A. Vergne.
Simplicial homology of random configurations.
Advances in Applied Probability., 2013.

A. Vergne, L. Decreusefond, and Ph. Martins.
Reduction algorithm for simplicial complexes.
In *Infocom*, April 2013.

F. Yan, Ph. Martins, and L. Decreusefond.
Connectivity-based distributed coverage hole detection in wireless sensor networks.
In *Globecom'11*, Houston, Texas, USA, August 2011.

F. Yan, Ph. Martins, and L. Decreusefond.
Accuracy of homology based approaches for coverage hole detection in wireless sensor networks.
In *ICC 2012*, June 2012.