Pricing Hedging de produits dérivés quels apports du pojet PREMIA

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16-11-2005

PREMIA

An option pricer

INRIA

ENPC

http://www.premia.fr

PREMIA: An Option Pricer

developped by the MathFi project

http://www-rocq.inria.fr/mathfi/

Joint activity of INRIA and CERMICS

- Provide C/C++ routines and scientific documentation for option pricing, hedging and model calibration.
- Objectives:
  - assist the R&D professional teams
  - help the academics to perform tests
  - provide graduate students with open-source examples

Consortium PREMIA: Members

Premia is developped in interaction with a consortium

Consortium Premia: functioning

- The participants of the consortium finance the development of Premia and help to determine the directions of future development.
- annual “delivery meating” + informal meetings
- Release $n-2$ “freeware” available on Premia web site

http://www.premia.fr

PREMIA : Description

- Software Component: C-programmed system designed to describe easily products, models and pricing methods
- Algorithm Component: each routine is written in a separate .c file
- Documention Component: hyperlinked PDF files which describe the routines and the mathematical methods (Monte Carlo, Tree methods. . . .)

PREMIA: Content

- The development of Premia started in 1999 and 7 versions have already been released.
- 170 algorithms for 16 different models have been implemented.
  For example, the American put in the Black-Scholes model can be priced using 33 different methods.

Releases 1-4

- Release 1,2 and 4: finite difference algorithms, trees and Monte Carlo methods for pricing and hedging standard and path-dependent European and American options on stocks in the Black-Scholes model, in 1 and 2 dimension.
- Release 3: Monte Carlo methods for American options in high dimension: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann

  Interface with Scilab software.

Releases 5-7

- Release 5 and 6: Quantization methods for American options,
  methods based on Malliavin calculus for European and American options,
  pricing, hedging and calibration algorithms for some models with jumps, local volatility, and stochastic volatility.
- Release 7: routines for pricing interest rate derivatives in some HJM and BGM models.
  calibration algorithms for various models (including stochastic volatility and jumps)
  methods based on Malliavin calculus for jump processes.

Release 8 (January 2006)

- Lévy models : Merton ,Variance Gamma, Tempered Stable, Kou, NIG
finite difference and Fourier transform-based routines for pricing and hedging european and barrier options .
Algorithm for non-parametric calibration of finite-intensity exponential Lévy models.
- Interest Rate Derivatives: affine models, Jump, Stochastic Volatility, CEV Diffusion Libor Model, Markov functional Libor market model
- Credit Derivatives: reduced-form models for pricing contingent claims subject to default risk (CDS, CDO).

Premia evolution

On a longer time scale Premia will evolve towards

- Risk Management including option hedging in interest rate and credit derivatives.
- Calibration

- Calibration in interest rate models (a data base of swaption and cap implied volatility has been provided by CDC).
- Calibration in credit risk model. (using e.g. credit defaut swap data)
- Calibration in incomplete market including stochastic volatility and/or jumps.
- Link between calibration and hedging in incomplete markets using liquid options.

- Pricing Energy derivatives.

Examples of methods

Calcul d'options américaines

$S_t \in \mathbb{R}^d$: prix sous-jacent

$$dS_t = b(t, X_t)dt + \sigma(t, X_t)dW_t, \;\; S_0 = x$$

Prix d'une option américaine, maturité T, cashflow $\phi(S_t)$.

$$v(t, S_t) = \sup_{\tau \in \mathcal{T}_{t,T}} \mathbb{E}(e^{-r(\tau - t)}\phi(S_\tau)|\mathcal{F}_t)$$

$$\max\left(\frac{\partial v}{\partial t}(t, x) + \mathcal{L}v(t, x) - rv(t, x), v - \phi\right) = 0$$

$$v(T, x) = \phi(x)$$

$$\mathcal{L}v = \frac{1}{2}\sum_{i,j=1}^{d}(\sigma\sigma^T)_{ij}\frac{\partial^2 v}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(x)\frac{\partial P}{\partial x_i}$$

Différences finies

Approximation de v aux points d'une grille Ω.
Par exemple, un θ-shéma implicite conduit à résoudre à chaque pas de temps une équation du type

$$v(x) = \max\left(\frac{1}{1+\rho} P v(x) + \ell(x), \phi(x)\right),$$

où $\rho < 1$ et P est une matrice Markovienne.

Algorithme d'Howard

Pour résoudre l'equation discrète $v(x) = \max\left(\frac{1}{1+\rho}Pv(x) + \ell(x), \phi(x)\right)$, on construit 2 suites (v^n) et $(\tilde{U}^n) : \Omega \to \{Continue, Stop\}$. Soit $\tilde{U}^0$ donné.

- *Etape* $2n-1$. Etant donné $\tilde{U}^n$, calculer v^n solution de

$$v^n(x) = \begin{cases} \frac{1}{1+r}Pv^n(x) + \ell(x) & \text{si } \tilde{U}^n(x) = Continue, \\ \phi(x) & \text{si } \tilde{U}^n(x) = Stop, \end{cases}$$

- *Etape* $2n$. Etant donné v^n, on définit $\tilde{U}^{n+1}$ par
  $\tilde{U}^{n+1}(x) = Continue$ si $\frac{1}{1+r}Pv^n(x) + \ell(x) > \phi$
  $\tilde{U}^{n+1}(x) = Stop$ sinon.

Principe du Maximum discret implique convergence.

Méthodes d'arbres

Basées sur approximation de la diffusion S_t par dynamique discrète. (Approximation par chaînes de Markov, e.g., arbre binomial)

Les méthodes d'EDP ou d'arbres sont inefficaces en dimension grande car en général les grilles en espace sont construites sans tenir compte des distributions des prix des actifs.

Méthodes de Monte Carlo

Basées sur des algorithmes de programmation dynamique pour la fonction valeur ou le temps d'arret optimal.
Soit $0 = t_0 < t_1 < \ldots < t_n = T$; $h = \frac{T}{n}$. option bermudéenne
Soit $(\bar{S}_{t_k}, k = 0 \ldots n)$ une approximation de $S_t, t \in [0, T]$.
Alors $P(t_k, \bar{S}_{t_k}) \sim \bar{P}_k$ défini par

$$\begin{aligned} \bar{P}_n &= \phi(\bar{S}_{t_n}) \\ \bar{P}_k &= \max(\phi(\bar{S}_{t_k}), e^{-rh}\mathbb{E}(\bar{P}_{k+1}|\bar{S}_{t_k})) \;\; 0 \leq k \leq n-1. \end{aligned}$$

On peut aussi calculer le delta $\delta(t, x) = \partial_x P(t, x)$.
Pb: évaluation numérique d'espérances conditionnelles par méthodes de Monte Carlo.

Algorithmes

- **Méthodes de régression** (projection sur des bases d'espaces d'Hilbert) pour l'approximation de l'espérance conditionnelle (Longstaff-Schwarz, Tsitsiklis-VanRoy)
- **Algorithmes de quantification** : (Bally- Pagès-Printems) les grilles tiennent compte de la loi du processus
- **Calcul de Malliavin** : Espérances conditionnelles représentées comme quotients d'espérances et calculées par Monte Carlo (Lions-Regner, Bally-Caramellino-Zanette)

$$\mathbb{E}(F(X_t)|X_s = \alpha) = \frac{\mathbb{E}(F(X_t)\pi_s^\alpha)}{\mathbb{E}(\pi_s^\alpha)}$$

π_s^α : poids
Techniques de réduction de variance par méthodes de localisation