SUSTAINABILITY OF FISHERIES MANAGEMENT AND VIABLE CONTROL OF DISCRETE TIME SYSTEMS

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Credits

- ACI Écologie quantitative MOOREA (Méthodes et outils d'optimisation pour la recherche en écologie appliquée)
- M. De Lara., L. Doyen, T. Guilbaud, M.-J. Rochet, *Is a management framework based on spawning stock biomass indicators sustainable? A viability approach*, to be published in *ICES Journal of Marine Science*. "high science content", "very relevant"
- M. De Lara., L. Doyen, T. Guilbaud, M.-J. Rochet, *Monotonicity properties for the viable control of discrete time systems*, *Systems and Control Letters*, Volume 56, Number 4, 2006, Pages 296-302.

Outline of the presentation

SUSTAINABLE MANAGEMENT

OF FISHERIES:

THE INTERNATIONAL COUNCIL FOR

THE EXPLORATION OF THE SEA (ICES)

RECOMMANDATIONS

Current fisheries management advice

Indicators and their associated **reference points** are key elements of current fisheries management advice, in the **International Council for the Exploration of the Sea** (**ICES**) precautionary approach.

The **Study Group for long term advice** is

- keeping (or restoring) **spawning stock biomass** SSB **indicator above a threshold reference point** B_{lim};
- restricting fishing effort so that **mean fishing mortality** F **indicator is below a threshold reference point** F_{lim}.

Discrete time nonlinear control system

$$\begin{cases} x(t+1) = g(x(t), u(t)), \quad t = t_0, t_0+1, \dots \\ x(t_0) \quad \text{given}, \end{cases}$$

where

- the *state variable* $x(t)$ belongs to the finite dimensional state space $\mathbb{X} = \mathbb{R}^{n_{\mathbb{X}}}$;
- the *control variable* $u(t)$ is an element of the *control set* $\mathbb{U} = \mathbb{R}^{n_{\mathbb{U}}}$;
- the *dynamics* g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Harvested fish population age structured model

- *Time* index t in years
- *State* variable $N = (N_1, \ldots, N_A) \in \mathbb{X} = \mathbb{R}_+^A$, the *abundances*-at-ages ($A = 3$ for anchovy and $A = 8$ for hake)
- *Control* variable $\lambda \in \mathbb{U} = \mathbb{R}_+$, *multiplier* of the *exploitation pattern* $F_1, \ldots, F_A$
- *Dynamics* g

Dynamics monotonicity

$$\begin{cases} g_1(N,\lambda) &= \varphi(SSB(N)), \\ g_a(N,\lambda) &= e^{-(M_{a-1}+\lambda F_{a-1})} N_{a-1}, \quad a = 2, \dots, A-1 \\ g_A(N,\lambda) &= e^{-(M_{A-1}+\lambda F_{A-1})} N_{A-1} + \pi \times e^{-(M_A+\lambda F_A)} N_A \,. \end{cases}$$

$g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is *decreasing with respect to the control*

$$\forall (N,\lambda) \in \mathbb{X} \times \mathbb{U} \quad \lambda' \geq \lambda \Rightarrow g(N,\lambda') \leq g(N,\lambda).$$

One year older every year...

Except for the recruits ($a = 1$) and the last age class ($a = A$),

$$N_{a+1}(t+1) = e^{-(M_a + \lambda(t) F_a)} N_a(t), \qquad \text{where}$$

- M_a stands for the *natural mortality-at-age a*;
- F_a is the *exploitation pattern-at-age a*;
- λ is the *exploitation pattern multiplier*.

The abundance of fishes of age greater than A verifies

$$N_A(t+1) = e^{-(M_{A-1} + \lambda(t) F_{A-1})} N_{A-1}(t) + \pi \times e^{-(M_A + \lambda(t) F_A)} N_A(t) \,.$$

Stock-recruitment

The recruits are

$$N_1(t+1) = \varphi(SSB(N(t)))$$

- $SSB(N)$ is the *spawning stock biomass*,

$$SSB(N) = \sum_{a=1}^{A} \gamma_a w_a N_a$$

  with γ_a proportion of matures-at-age a
  and w_a weight-at-age a,
- φ describes a **stock-recruitment relationship**.

Uncertainty on stock-recruitment relationship

Recruitment involves complex biological and environmental processes that fluctuate in time, and are difficult to integrate into a population model.

Typical examples are

- constant: $\varphi(B) = R$;
- linear: $\varphi(B) = rB$;
- Beverton-Holt: $\varphi(B) = \frac{B}{\alpha+\beta B}$;
- Ricker: $\varphi(B) = \alpha B \mathrm{e}^{-\beta B}$.

The ICES indicators and reference points

- **Spawning stock biomass**,

$$SSB(N) = \sum_{a=1}^{A} \gamma_a w_a N_a$$

with reference threshold $SSB(N) \geq B_{\lim}$.

- **Mean fishing mortality** over a pre-determined age range from a_r to A_r, that is,

$$F(\lambda) := \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{a=A_r} F_a$$

with reference threshold $F(\lambda) \leq F_{\lim}$.

ACCEPTABLE CONFIGURATIONS AND VIABILITY DOMAINS

Acceptable configurations

We introduce a subset

$$\mathbb{D} \subset \mathbb{X} \times \mathbb{U} = \quad \text{“states”} \quad \times \quad \text{“controls”}$$

termed the **acceptable configurations set**.

We aim at finding at least one trajectory such that

$$(x(t), u(t)) \in \mathbb{D}, \quad t = t_0, t_0 + 1, \ldots$$

Examples of acceptable configurations sets

Considering sustainable management within the PA, involving SSB and F indicators, we introduce the following PA **configuration set**:

$$\mathbb{D}_{\text{lim}} := \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\text{lim}} \text{ and } F(\lambda) \leq F_{\text{lim}}\}.$$

Viability domains

Definition

A subset $\mathbb{V} \subset \mathbb{X}$ is said to be a **viability domain** for the dynamics g in the acceptable set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ if

$$\forall x \in \mathbb{V}\,, \quad \exists u \in \mathbb{U}\,, \quad (x, u) \in \mathbb{D} \text{ and } g(x, u) \in \mathbb{V}.$$

Viable controls

When $\mathbb{V}$ is a viability domain, the following set of controls is not empty:

$$\mathbb{U}_{\mathbb{V}}(x) = \{u \in \mathbb{U} \mid (x,u) \in \mathbb{D} \text{ and } g(x,u) \in \mathbb{V}\}.$$

Definition

A **viable policy** is a mapping $\Psi : \mathbb{X} \to \mathbb{U}$ which associates with each state $x \in \mathbb{V}$ a control $u = \Psi(x)$ satisfying $\Psi(x) \in \mathbb{U}_{\mathbb{V}}(x)$.

Starting from $x(t_0) \in \mathbb{V}$ and applying a viable policy $u(t) = \Psi(x(t))$ yields a trajectory satisfying

$$(x(t), u(t)) \in \mathbb{D}, \quad t = t_0, t_0+1, \ldots$$

Comments on ICES precautionary approach

The precautionary approach (PA) may be sketched as follows:

- the condition $SSB(N) \geq B_{\lim}$ is checked;
- if valid, the following usual advice is given:

$$\lambda_{UA}(N) = \max\{\lambda \in \mathbb{R}_+ \mid SSB(g(N,\lambda)) \geq B_{\lim} \text{ and } F(\lambda) \leq F_{\lim}\}$$

The problem is that... **nothing ensures the existence of $\lambda \geq 0$ such that $SSB(g(N,\lambda)) \geq B_{\lim}$ and $F(\lambda) \leq F_{\lim}$.**

The existence of a fishing mortality multiplier for any stock vector N such that $SSB(N) \geq B_{\lim}$ is **tantamount to non-emptyness of a set of viable controls**. This justifies the following definitions.

ARE THE ICES RECOMMANDATIONS

"SUSTAINABLE"?

Defining "sustainability"

Let us define the PA state set

$$\mathbb{V}_{\text{lim}} := \{N \in \mathbb{R}_+^A \mid SSB(N) \geq B_{\text{lim}}\}.$$

We shall say that **the precautionary approach is sustainable** if the PA state set $\mathbb{V}_{\text{lim}}$ is a viability domain for dynamics g in the acceptable set

$$\mathbb{D}_{\text{lim}} = \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\text{lim}} \text{ and } F(\lambda) \leq F_{\text{lim}}\}.$$

Indeed, starting from $N(t_0) \in \mathbb{V}_{\text{lim}}$ and applying the usual policy λ_{UA} yields a trajectory satisfying

$$SSB(N(t)) \geq B_{\text{lim}} \text{ and } F(\lambda(t)) \leq F_{\text{lim}}, \quad \forall t = t_0, t_0 + 1, \ldots$$

Testing "sustainability"

Proposition

The PA *is sustainable if and only if*

$$\min_{B \in [B_{\lim}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\lim}$$

that is, if and only if the lowest possible sum of survivors (weighted by growth and maturation) and newly recruited spawning biomass is above $B_{\lim}$*.*

$$\Theta = \min \left(\min_{a=1,\dots,A-1,\ \gamma_a w_a \neq 0} \left[\frac{\gamma_{a+1} w_{a+1}}{\gamma_a w_a} e^{-M_a} \right], \pi e^{-M_A} \right)$$

The proof relies upon *monotonicity properties*.

The answer depends upon... the stock-recruitment relationship φ.
Notice that condition

$$\min_{B \in [B_{\mathrm{lim}}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\mathrm{lim}}$$

does not depend on the stock-recruitment relationship φ between 0 and B_{lim}.
It does not depend on F_{lim} either.

Testing "sustainability" with constant recruitment

A constant recruitment is generally used for fishing advice, so the following simplified condition can be used.

If we suppose that

- the natural mortality is independent of age, that is $M_a = M$,
- the proportion γ_a of mature individuals and the weight w_a at-age are increasing with age a,
- the stock-recruitment is a constant R,

the PA is sustainable if and only if

$$R \geq \underline{R} \quad \text{where} \quad \underline{R} := \frac{1 - \pi e^{-M}}{\gamma_1 w_1} B_{\text{lim}} ,$$

making thus of $\underline{R}$ a minimum recruitment required to preserve B_{lim}.

Bay of Biscay anchovy

S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)	Linear	Ricker
Condition to check	$R_{\text{mean}} \geq \underline{R}$	$R_{\text{gm}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$\gamma_1 w_1 r \geq 1$	$\min_{B \geq B_{\text{lim}}} [\cdots] \geq$
Left hand side	$14\ 016 \times 10^6$	$7\ 109 \times 10^6$	$3\ 964 \times 10^6$	696×10^6	0.84	0
Right hand side	$1\ 312 \times 10^6$	$1\ 312 \times 10^6$	$1\ 312 \times 10^6$	$1\ 312 \times 10^6$	1	21 000
Sustainable	yes	yes	yes	no	no	no

Table: Bay of Biscay anchovy: sustainability of advice based on the spawning stock biomass indicator for various stock recruitment relationships. The answer is given in the last row of the table. The second row contains an expression whose value is given in the third line. It has to be compared to the threshold in the fourth row.

Northern stock of hake

For hake, ICES precautionary approach is never sustainable because the proportion of mature individuals at age 1 is zero, $\gamma_1 = 0$, so condition

$$\min_{B \in [B_{\lim}, +\infty[} [\Theta B + \gamma_1 w_1 \varphi(B)] \geq B_{\lim}$$

is never satisfied, whatever the value of $B_{\lim}$. Indeed, $\Theta \leq \pi e^{-M_A} < 1$ since $\pi \in \{0, 1\}$ and $M_A > 0$.

Thus, **a viability domain based upon the only indicator *SSB* proves unsufficient**.

A confusion due to the double role of SSB indicator

The *SSB* indicator is used for two different purposes:

- for designing *short term advice*: when $SSB(N(t)) \geq B_{\text{lim}}$ is checked, compute usual advice $\lambda_{UA}(N(t)) = \max\{\lambda \in \mathbb{R}_+ \mid SSB(g(N(t),\lambda)) \geq B_{\text{lim}} \text{ and } F(\lambda) \leq F_{\text{lim}}\}$;
- for delineating a domain to which states and controls should belong *year after year*:

$$SSB(N(t)) \geq B_{\text{lim}} \text{ and } F(\lambda(t)) \leq F_{\text{lim}}\,, \quad \forall t = t_0, t_0+1, \dots$$

There is no reason why yearly objectives described by means of the single *SSB* indicator should be achieved by means of advice based upon this single indicator.

HOW TO DESIGN "SUSTAINABLE"

RECOMMANDATIONS

Viability kernel

Definition

The following set of states

$$\mathbb{V}(g,\mathbb{D}) := \left\{ x(0) \in \mathbb{X} \;\middle|\; \begin{array}{l} \exists\ (u(0),u(1),\ldots) \text{ and } (x(0),x(1),\ldots) \\ \text{satisfying } x(t+1) = g(x(t),u(t)) \\ \text{and } (x(t),u(t)) \in \mathbb{D}, \quad t \geq 0 \end{array} \right\}$$

is called the **viability kernel** associated with the dynamics g in the acceptable set $\mathbb{D}$.

Starting from $x(0) \in \mathbb{V}(g,\mathbb{D})$, there exists a sequence of states and of controls which both satisfy the dynamics g and belong to the acceptable set $\mathbb{D}$.

The viability kernel $\mathbb{V}(g,\mathbb{D})$ turns out to be the **union of all viability domains**.

The "comfortable" case

The *state constraints set* associated with $\mathbb{D}$ is obtained by projecting the acceptable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \mathrm{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,, \quad (x, u) \in \mathbb{D}\}.$$

Notice that

$$\mathbb{V}(g, \mathbb{D}) \subset \mathbb{V}^0\,.$$

The **comfortable case** is when

$$\mathbb{V}(g, \mathbb{D}) = \mathbb{V}^0\,.$$

This is the implicit assumption in ICES precautionary approach.

Sustainable management

Definition

We say that **sustainable management is possible within** ICES **bounds** if the viability kernel $\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}})$ associated with dynamics g in the acceptable set

$$\mathbb{D}_{\mathrm{lim}} = \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid SSB(N) \geq B_{\mathrm{lim}} \text{ and } F(\lambda) \leq F_{\mathrm{lim}}\}$$

is not empty.

Proposition

$$\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}}) = \left\{ N \in \mathbb{R}_+^A \,\middle|\, SSB\left(g^{(n)}(N, 0)\right) \geq B_{\mathrm{lim}} \quad \forall n \right\},$$

with $g^{(n)}(\cdot, 0) = g(\cdot, 0) \circ g(\cdot, 0) \circ \cdots \circ g(\cdot, 0)$, the n-time composition of the mapping $g(\cdot, 0) : \mathbb{R}_+^A \to \mathbb{R}_+^A$.

Constant recruitment and no plus-group

Proposition

With a constant recruitment R and no plus-group ($\pi = 0$), the set $\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}})$ is described by $A + 1$ affine constraints:

$$\mathrm{Viab}(g, \mathbb{D}_{\mathrm{lim}}) = \Big\{ N \in \mathbb{R}_+^A \mid \quad R\,\mathrm{spr}(0) \geq B_{\mathrm{lim}} \quad \textit{and} \quad SSB\left(g^{(i)}(N, 0)\right) \geq B_{\mathrm{lim}} \quad \forall i = 0, \dots, A \Big\} .$$

Constant recruitment and no plus-group

Proposition

Sustainability is characterized as follows:

$$\mathrm{Viab}(g, \mathbb{D}_{\mathsf{lim}}) \neq \emptyset \quad \Leftrightarrow \quad R\, \mathsf{spr}(0) \geq B_{\mathsf{lim}}.$$

The well known *spawners per recruit* indicator appears naturally. We denote by $\mathsf{spr}(\lambda)$ the equilibrium *spawners per recruit* obtained with the fishing mortality multiplier λ. By definition,

$$\mathsf{spr}(\lambda) = \frac{SSB(\overline{N})}{\varphi(SSB(\overline{N}))} \quad \text{where} \quad \overline{N} = g(\overline{N}, \lambda).$$

Bay of Biscay anchovy

S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)
Condition	$R_{\text{mean}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{gm}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{min}}\,\text{spr}(0) \geq B_{\text{lim}}$	$R_{\text{min}}\,\text{spr}(0) \geq B_{\text{lim}}$
Left hand side	194.1×10^6	98.5×10^6	54.9×10^6	9.6×10^6
Right hand side	21×10^6	21×10^6	21×10^6	21×10^6
sustainable management	yes	yes	yes	no

Table: Bay of Biscay anchovy: Sustainable management for some stock recruitment constant relationships.

For stock recruitment constant R such that

$$1\ 312 \times 10^6 \leq R \leq 1\ 516 \times 10^6$$

sustainable management is possible but . . .
not following ICES advice! New indicators, others than SSB.

Coping with uncertain dynamics in a precautionary way

For a stock-recruitment relationship

$$\varphi^\flat \leq \varphi$$

and the associated dynamics

$$g^\flat \leq g$$

we have

$$\mathrm{Viab}(g^\flat, \mathbb{D}_{\mathsf{lim}}) \subset \mathrm{Viab}(g, \mathbb{D}_{\mathsf{lim}}) .$$

Hence, by choosing a constant mimimum recruitment, we obtain a lower approximation for all majorizing dynamics.

CONCLUSIONS

Claims about sustainable management

Although **sustainable management** is claimed to be a guide for decision making, there is great **confusion** between

- **operational objectives (advice)**
- and **perpetual objectives** (not explicitely stated).

Viability concepts and methods have helped

- giving a framework for **setting decision making**;
- **testing** "sutainability" of current fishing advice and practices;
- **proposing** viable policies;

giving thus coherence to claims and practices.

Perspectives

The approach developed

- relies upon monotonicity properties of viability domains with respect to the dynamics and to the acceptable set;
- may be extended to multiple species *without ecological interactions but with technical interactions*;
- may include explicit economic requirements such as mimimum yield

$$\mathbb{D}_{\text{yield}} := \{(N, \lambda) \in \mathbb{R}_+^A \times \mathbb{R}_+ \mid Y(N, \lambda) \geq y_{\mathsf{min}}\}.$$

Perspectives

Mathématiques et décision pour le développement durable
RTP M3D
Proposition de *Réseau thématique prioritaire* CNRS

- département Environnement et développement durable
- département Sciences humaines et sociales
- département Mathématiques, physique, planète et univers