Overcoming the course of dimensionality with DNNs: Theoretical approximation results for PDEs

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Artificial neural networks (ANNs) have very successfully been used in numerical simulations for a series of computational problems ranging from image classification to numerical approximations of partial differential equations (PDEs). Such numerical simulations suggest that ANNs have the capacity to very efficiently approximate high-dimensional functions and, especially, such numerical simulations indicate that ANNs seem to admit the fundamental power to overcome the curse of dimensionality when approximating the high-dimensional functions appearing in the above named computational problems. Although there are numerous results on approximation capacities of ANNs such as, e.g., the universal approximation theorem, most of them cannot explain the empirical success of ANNs when approximating high-dimensional functions. In this talk I will explain recent theoretical developments which demonstrate that ANNs can efficiently approximate solutions of high-dimensional PDEs. More precisely, I will present results revealing that the minimal required number of parameters of an ANN to approximate solutions of certain PDEs grows at most polynomially in both the reciprocal $1/\varepsilon$ of the prescribed approximation accuracy $\varepsilon > 0$ and the PDE dimension $d \in \mathbb{N}$. Those statements prove that ANNs do indeed have the capacity to overcome the curse of dimensionality in the numerical approximation of PDEs.

Références

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