On exact polynomial optimization through sums of squares, sums of nonnegative circuits and sums of AM/GM-exponentials

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Mots-clés : nonnegative circuit polynomial, arithmetic-geometric-mean exponential, convex optimization, geometric programming, relative entropy programming, semidefinite programming, exact certificate, rounding-projection procedure, sums of squares decomposition, Putinar's representation, hybrid numeric-symbolic algorithm, real algebraic geometry

We consider the problem of certifying nonnegativity for certain classes of multivariate polynomials. We focus on finding exact sums of squares (SOS), sums of nonnegative circuits (SONC) and sums of arithmetic-geometric-mean-exponentials (SAGE) decompositions. For each case, we rely on existing relaxations: semidefinite programming (SDP) for SOS decompositions, geometric programming (GP) for SONC decompositions and relative entropy programming (REP) for SAGE decompositions.
For the SOS case, we provide a hybrid numeric-symbolic algorithm [1] computing exact rational decompositions for polynomials lying in the interior of the SOS cone. It computes an approximate SOS decomposition for a perturbation of the input polynomial with an arbitrary-precision SDP solver. An exact SOS decomposition is obtained thanks to the perturbation terms.
We also provide two hybrid numeric-symbolic optimization algorithms [2], computing exact SONC and SAGE decompositions. Moreover, we provide a hybrid numeric-symbolic decision algorithm for polynomials lying in the interior of the SAGE cone. Each framework, inspired by previous contributions of Parrilo and Peyrl, is a rounding-projection procedure.
For a polynomial lying in the interior of the SOS or SAGE cone, we prove that the corresponding decision algorithm terminates within a number of arithmetic operations, which is polynomial in the degree and number of terms of the input, and singly exponential in the number of variables.
We also provide experimental comparisons between the implementation of these algorithms and available alternatives.

Références

[1] V. Magron and M. Safey El Din. On Exact Polya, Hilbert-Artin and Putinar's Representations. arXiv:1811.10062, 2018.

[2] V. Magron, H. Seidler and T. de Wolff. Exact optimization via sums of nonnegative circuits and sums of AM/GM exponentials. arXiv:1902.02123, 2019.

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