Energy methods in continuous and discrete time

Tomas Lundquist, Université Toulouse III Paul Sabatier

For the design and analysis of stable boundary procedures of hyperbolic problems, some of the most widely successful and generally applicable strategies involve the use of energy methods. By adopting a discrete formalism centered on achieving key properties such as integration-by-parts, energy proofs of well-posedness can be mimicked more or less exactly in order to prove stability of a numerical scheme. For this to be possible, the discrete operations of integration and differentiation must be tightly coupled, and often special care must be taken when imposing physical or artificial boundary conditions. In recent years, operators with a formal summation-by-parts property have been used with great success to fashion provably stable discretizations on complex geometries involving e.g. numerical boundary conditions, multi-block or hybrid interface couplings, curvilinear transformations and moving meshes. This emerging summation-by-parts framework has so far been exclusively focused on method-of-lines type techniques, where it can be applied to a wide range of numerical methods including finite element, finite difference and finite volume methods. In this talk we review some of the latest developments in energy stable numerical techniques, including generalized definitions of summation-by-parts operators, weak boundary/interface conditions and numerical filters. We will also discuss challenges and possible benefits from extending the summation-by-parts idea to discrete time, with particular focus on schemes of Lax-Friedrichs or Lax-Wendroff type.

Tomas Lundquist, Université Toulouse III Paul Sabatier, 118 route de Narbonne, 31062 Toulouse
`Tomas.Lundquist@math.univ-toulouse.fr`