Stability of numerical schemes for the hyperbolic relaxation system with boundary *

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In many physical situations, we are interested in hyperbolic systems of partial differential equations with relaxation terms. Such systems are found in relaxing gas theory, water waves and reactive flows. One of the main features of these models is related to the notion of dissipation, leading to smooth solutions and asymptotic stability. The most classical model is the damped wave equation and we consider here the associated initial boundary value problem (IBVP) in the quarter plane. This problem has been addressed by Xin and Xu who derive a necessary and sufficient condition for stiff stability in [1], i.e. stability uniformly with respect to the stiffness of the relaxation term. Our aim is to study the discrete case in the context of finite difference or finite volume approximations [2][3]. We study for instance the relationship between the stiff stability of numerical solutions and the Stiff Kreiss Condition (SKC) for the boundary problem. Due to the effects of the boundary layer and the interactions of the boundary and initial layer, numerical schemes have to be properly designed in order to provide accurate approximations and consistent behaviors. The asymptotic stability and boundary layer behavior are studied by summation by parts operators, energy estimate and L2 analysis [3].

* This is a joint work under supervise of Professors Nicolas SEGUIN (IRMAR, University of Rennes 1) and Benjamin BOUTIN (IRMAR, University of Rennes 1).

Références

[1] Zhouping Xin and Wen-Qing Xu, *Stiff well-posedness and asymptotic convergence for a class of linear relaxation systems in a quarter plane*, J. Differential Equations, 167(2):388437, 2000.

[2] Shi Jin and Zhou Ping Xin, *The relaxation schemes for systems of con- servation laws in arbitrary space dimensions*,Comm. Pure Appl. Math., 48(3):235276, 1995.

[3] B. Gustafsson, H.O. Kreiss, J. Oliger, *Time dependent problems and difference methods*,Wiley, 2013.

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