When Sally Found Harry: A Stochastic Search Game

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We study a hide-search model where a hider (Harry) hides on an edge of a graph and a searcher (Sally) travels around the graph in search of Harry. Her goal is to find him as soon as possible.
The novelty of the model is that, due to various circumstances, at any given time, some edges may be unavailable, so the graph randomly evolves over time. At each stage, each edge e of the graph is, independently of the others, active with probability p_e and inactive with probability $1 - p_e$.
At the beginning of the game, Harry chooses one edge to hide in and is immobile for the rest of the game. Starting from an initial vertex called the root of the graph, Sally chooses at each stage a vertex among those active in the neighborhood of her current vertex. The game ends when Sally traverses the edge where Harry is hidden, and the payoff is the number of the stage at which the game ends. So, Harry aims at maximizing the time by which he is found by Sally, while Sally tries to minimize this time. This can be modeled as a zero-sum two-person game.
We first examine the deterministic version of the game when p_e equals 1 for each edge e. By adapting some well-known results in continuous time, we prove that this game has a value and provide an upper and lower bound for this value: they correspond, for a fixed number of edges, to the values of the games played on a tree or an Eulerian graph.
We then turn to the stochastic case and we show that, even in this case, the game has a value for all positive p_e. We provide an upper and lower bound for this value and show that it converges to the value of the deterministic game when p_e goes to 1 for each edge e. Finally, we consider some particular instances when all p_e are equal and study the two extreme classes of graphs that are trees and Eulerian graphs. We generalize the optimal strategies of the deterministic version to the stochastic case and obtain bounds which happen to be tight for some subclasses of graphs. .

Références

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