On first order elliptic systems

Michaël NDJINGA, CEA Saclay

Sédrick KAMENI NGWAMOU, Université de Yaoundé I

Mots-clés : Elliptic systems, Friedrichs' systems, Poisson equation, weak formulation, Fourier transform, Banach-Necas-Babuska theorem

We are interested in elliptic systems written in the form of first order PDE systems.

$$\sum_{k=1}^{d} A_k \partial_k U + KU = F \text{ on } \partial\Omega, \tag{1}$$

There are several definitions of elliptic systems [5, 4, **?**, 3]. This is due to the fact that the simplest extensions of the scalar definition do not enjoy the property that an elliptic system remains elliptic under a change of variables.

The classical theory ([7, 9, 8, 10]) of Friedrichs' systems makes some assumptions such as $K > 0$ that enable the use of the Lax-Milgram theorem to derive existence results (see for instance theorem 5.4 in [1]). However, the assumption $K > 0$ is a serious obstacle to the analysis of linear symmetric hyperbolic systems in the stationary regime since these do not necessarily possess a friction operator K, especially they represent conservation laws. We also remark that in the case $\Omega = \mathbb{R}^d$, Fourier analysis has shows that the assumption $K > 0$ is not necessary but only sufficient for the existence and uniqueness of solutions. Furthermore we remark that the assumption $K > 0$ does not allow the analysis of the first order reduction of the Poisson problem where the kernel of K is not trivial. We therefore look for an alternative theory based on weaker assumptions. We use the more general Banach-Nečas-Babuška theorem to obtain the existence of a uniqueness of a solution in a setting that encompasses both Friedrichs' systems and the first order reduction of the Poisson problem. The techniques used to prove the classical inf-sup conditions inspired from harmonic analysis arguments that are consistent with the case $\Omega = \mathbb{R}^d$.

Références

[1] Ern, Alexandre, and Jean-Luc Guermond. Theory and practice of finite elements. Vol. 159. Springer Science & Business Media, 2013.

[2] Haïm Brezis, "Functional Analysis, Sobolev Spaces and PDEs", Springer, 2010

[3] Krupchyk, Katsiaryna, Werner M. Seiler, and Jukka Tuomela. "Overdetermined elliptic systems." Foundations of Computational Mathematics 6.3 (2006): 309-351.

[4] Cosner, Chris, On the definition of ellipticity for systems of partial differential equations, Journal of Mathematical Analysis and Applications, vol 158, 1991

[5] Bochev, Pavel B., Gunzburger, Max D., Least-squares finite element methods, Springer, 2009

[6] Agmon, S., Douglis, A., Nirenberg, L., Estimates near the boundary for solutions of elliptic partial differential equations satisfying general boundary conditions II, Comm. Pure Appl. Math., 1964

[7] K. O. Friedrichs: Symmetric positive linear differential equations, Communications on Pure and Applied Mathematics 11 (1958), 333418

[8] A. Ern, J.-L. Guermond, Discontinuous Galerkin methods for Friedrichs systems. I. General theory, SIAM J. Numer. Anal., 44 (2006), pp. 753778

[9] A. Ern, J.-L. Guermond, G. Caplain, An Intrinsic Criterion for the Bijectivity Of Hilbert Operators Related to Friedrichs Systems, Comm. Partial Diff. Eq., 32 (2007), 317341

[10] Nenad Antonić, Krešimir Burazin, Ivana Crnjac, Marko Erceg, Complex Friedrichs systems and applications, Journal of Mathematical Physics 58(101508)

Michaël NDJINGA, Université Paris-Saclay, CEA Saclay, DEN/DANS/DM2SSTMF/LMEC, 91191 Gif-sur-Yvette
michael.ndjinga@cea.fr