An operator splitting method for the time discretization of a multi-scale model in ophthalmology

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Mots-clés : partial and ordinary differential equations, multi-scale model, operator splitting, hemodynamics, ocular blood flow, porous media

Glaucoma is a neuro-degenerative disease that involves the optic nerve head and it is one of main cause of blindness. The challenges to face in the development of an *in silico* model for this neuropathology are the multi-scale and multi-physics biological mechanisms of the human eye, in particular the need of hemodynamical information within the ocular posterior vasculature that is not easily accessible with standard clinical measurements.
To this end, we propose a mathematical and computational framework called the Ocular Mathematical Virtual Simulator (OMVS) [1], which couples (i) the hemodynamics of the ocular posterior tissue, described by a three-dimensional poro-elastic system and (ii) the blood circulation in the retinal vasculature and central retinal vessels represented by a lumped-parameter circuit that exploits the electric analogy to fluid flow. In this geometric multi-scale model, in order to solve system (i) of Partial Differential Equations (PDEs) we have adopted the Hybridizable Discontinuous Galerkin method with an Integral Interface Condition [2], interfacing it with system (ii) of Ordinary Differential Equations (ODEs) by means of an innovative operator splitting approach [3]. Such splitting design ensures that the physical energy balance is maintained at the discrete level and, as a consequence, unconditional stability is attained *via* appropriate initial condition for each sub-step without the need of sub-iterating. In addition, the modularity of this approach allows us to simulate mechanisms that have different temporal dynamics. Finally, motivated by an application in ophthalmology, we have formulated and implemented a coupling algorithm able to deal with different scales in space and time, which also provides a significant gain in terms of computational performances for the resulting overall system. Thus, this numerical method exhibits robustness and efficiency, properties that are essential not only to ensure physical solutions in Life Sciences, but also to solve mathematical challenges in other research fields.

Références

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