Parameter identification for a coupled Darcy-Stokes problem using the Unscented Kalman Filter

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Nowadays patient-specific biological models are of increasing interest. They find their application in high-quality prediction of treatment response or disease progression. Modeling and simulation of blood flows is an essential topic in mathematics applied to medicine. Our work aims to achieve modeling of patient-specific cerebral blood flow for the diagnostic and prognostic purpose. The patient-specificity arises in the geometry and the parameters of these models that are to be recovered from indirect measurements. We propose in the present contribution to lay the mathematical and computational framework to adjust and solve this class of models numerically, with a unifying implementation of fluid flow models and the corresponding parameter identification methods. The large vessels and brain tissue interaction occur through a virtual boundary between two domains. We propose a well-posed coupled Darcy-Stokes model for such interactions, with the experimental Beavers-Joseph-Saffman coupling condition [1], [2], [3]. This system requires an appropriate choice of discrete function spaces based on the finite element method [1] and the use of the hybrid discrete Galerkin (HDG) method [4], for the Darcy subproblem. We numerically solved this problem within the framework of the C++ library Feel++, more precisely via a Python wrapper to ease the interface with other software. The parameter estimation is performed using an iterative discrete process known as the Kalman filter. In particular, we implemented in Python the unscented Kalman filter (UKF) [5], which has a compliant behavior to the nonlinearity of the model thanks to the unscented transform by propagating the uncertainty as additional knowledge on the estimation. This property allowed us to perform accurate parameter estimation thanks to the joint state-parameter formulation.
We shall present numerical experiments of both Darcy-Stokes flows and parameter estimation situations separately and then coupled together on test cases relevant to our context using synthetic data.

Références

[1] M. Discacciati, A. Quarteroni, Navier-Stokes/Darcy Coupling : Modeling, Analysis, and Numerical Approximation, Rev. Mat. Comput. Vol. 22, no. 2, 315–426, 2009.

[2] J.M. Urquiza, D. N'Dri, A. Garon, M.C. Delfour, Coupling Stokes and Darcy equations, Applied Numerical Mathematics, Vol. 58, Issue 5, 525–538, 2008.

[3] G. N. Gatica, F. A. Sequeira, Analysis of an Augmented HDG Method for a Class of Quasi-Newtonian Stokes Flows, J. Sci. Comput., Vol. 65 Issue 3, 1270—1308, 2015.

[4] B. Cockburn, J. Gopalakrishnan, R. Lazarov, Unified hybridization of discontinuous Galerkin, mixed, and conforming Galerkin methods for second order elliptic problems, SIAM J. Numer. Anal., Vol. 47, 1319–1365, 2009.

[5] E.A. Wan, R. van der Merwe, The Unscented Kalman Filter for Nonlinear Estimation, Oregon Graduate Institute of Science & Technology, 2000.

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