Modeling underground flows in shallow aquifers

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In this work, we present a class of models describing water flow in shallow aquifers (with a small deepness with respect to their horizontal length). Each model of the class is coupling 1D vertical Richards equations with 2D horizontal Dupuit problem (same kind of models are proposed in [1, 2]). They are approximations of the classical 3D-Richards system which is often used to characterize the flow in porous soil. These models are based on the dominant behaviors of the flow in shallow aquifers depending on the considered time-scale. In a short-time scale, the dominant flow turns to be vertical and mainly occurs in the unsaturated part of aquifer (upper part). In the other hand, for the long-time scale, the vertical flow appears to be instantaneous and the horizontal one become significant. In practice, the soil is splitted into two parts by a function $z = h(t, x)$ and the flow is characterized in each part as follows.

In the upper part: The flow is assumed to be vertical and is described by the following 1D Richard's equation and Darcy's law

$$\frac{\partial s(P)}{\partial t} + \frac{\partial u}{\partial z} = 0, \qquad u = -k_r(P)\left(\frac{\partial P}{\partial z} + 1\right) e_3, \tag{1}$$

where $s(P) \in [0, 1]$ is the soil saturation and $k_r(P) \in [0, 1]$ the soil conductivity.

In the lower part: The vertical flow is instantaneous (this is the so-called Dupuit hypothesis). The corresponding pressure profile is affine in z and is characterized by the hydraulic head H (which is constant with respect to z). It is given as follows

$$P\big(t, x, z\big) = H(t, x) - z. \tag{2}$$

The characterization of H is given through the following bi-dimensionnal mass conservation equation

$$\mathrm{div}_x\Big(\tilde{k}(H)\,\nabla_x H\Big) = u\big|_h \cdot e_3. \tag{3}$$

where $\tilde{k}(H)$ is an averaged conductivity along vertical water columns and $u\big|_h \cdot e_3$ is a water flux at the interface $z = h$. The description of the model is closed by a general relation $h(t, x) = Q(x, H(t, x))$ which reduces in simple cases to $h(t, x) = H(t, x) - R$ for a given $R \geq 0$. As it is the case for the original 3d-Richards problem, this model turns to be mass-conservative.

Justification of the model. The idea is to compare the effective problems obtained when the aquifer admits a ratio $\varepsilon =$ *deepness/horizontal length* which tends to 0 (in different cases of time-scale). In practice we give the effective problems obtained from the 3d-Richards equations in short, intermediate and long time scale. Next, we do the same asymptotic analysis on the coupled model and show that the corresponding effective problems are exactly the same for every considered time scale.
Numerical simulations will be presented to compare the original 3d-Richards problem and the coupled problem.

Références

[1] Mary F Pikul, Robert L Street, and Irwin Remson, *A numerical model based on coupled one-dimensional richards and boussinesq equation.* Water Resources Research, 10(2):295-302, 1974.

[2] P. Sochala, A. Ern, and S. Piperno, *Mass conservative BDF-discontinuous Galerkin/explicit finite volume schemes for coupling subsurface and overland flows, Computer Methods in Applied Mechanics and Engineering.* vol.198, issue.27-29, pp.2122-2136, 2009.

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