# Guaranteed and robust $L^2$ a posteriori error estimates for 1D linear advection problems

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We propose a reconstruction-based a posteriori error estimation for linear advection problems in one-dimension. In our framework [1], a stable variational formulation for the ultra-weak solution is adopted, and the equivalence of the $L^2$-norm of the error and the dual graph norm of the residual is established. This dual norm is showed to be localizable on the patches over the domain under mild orthogonality with respect to hat functions. This condition is valid for some well-known numerical methods including continuous/discontinuous Petrov–Galekin and discontinuous Galerkin methods.
Consequently, a well-posed local problem on each two-cell patch centered around a mesh vertex is presented which gives a global reconstruction of the solution in the graph space. We prove that this reconstruction leads to a guaranteed upper bound and a local lower bound (up to a constant) of the error. The local efficiency is robust with respect to the mesh-refinement, the polynomial degree of the approximation and the advective velocity. These properties are verified in a series of numerical experiments.
Motivated by these results, we propose a heuristic extension to two-dimensional case. This is achieved by solving a local least-squares problem on each patch. The performance of this error indicator is numerically tested.

## Références

[1] A. Ern, M. Vohralik and M. Zakerzadeh, *Guaranteed and robust $L^2$ a posteriori error estimates for 1D linear advection problems*, In preparation.

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