Local asymptotic properties for Cox-Ingersoll-Ross process with discrete observations

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Mots-clés : Cox-Ingersoll-Ross process; local asymptotic (mixed) normality property; local asymptotic quadraticity property; Malliavin calculus; parametric estimation; square root coefficient.

We consider a Cox-Ingersoll-Ross process whose drift coefficient depends on unknown parameters. Considering the process discretely observed at high frequency, we prove the local asymptotic normality property in the subcritical case, the local asymptotic quadraticity in the critical case, and the local asymptotic mixed normality property in the supercritical case. To obtain these results, we use the Malliavin calculus techniques developed recently for CIR process by Alòs et *al.* [1] and Altmayer et *al.* [2] together with the L^p-norm estimation for positive and negative moments of the CIR process obtained by Bossy et *al.* [5] and Ben Alaya et *al.* [3, 4]. In this study, we require the same conditions of high frequency $\Delta_n \to 0$ and infinite horizon $n\Delta_n \to \infty$ as in the case of ergodic diffusions with globally Lipschitz coefficients studied earlier by Gobet [6]. However, in the non-ergodic cases, additional assumptions on the decreasing rate of Δ_n are required due to the fact that the square root diffusion coefficient of the CIR process is not regular enough. Indeed, we assume $\frac{n\Delta_n^{\frac{3}{2}}}{\log(n\Delta_n)} \to 0$ for the critical case and $n\Delta_n^2 \to 0$ for the supercritical case.

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