

Some quantitative homogenization results of an interface

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We consider a elliptic equation with a small scale $\epsilon \ll 1$

$$-\operatorname{div}(a(x/\epsilon) \cdot \nabla u^\epsilon(x)) = f(x),$$

where the matrix a represents a flat interface between two heterogeneous media. Remarkably enough, the homogenized matrix \bar{a} is not constant, but only piecewise constant and discontinuous through the interface. We introduce a generalization of the two-scale expansion to obtain a precise approximation of the oscillating gradient ∇u^ϵ . Then, by combining Avellaneda and Lin's work in homogenization and Li and Vogelius' work concerning regularity of elliptic equation with discontinuous coefficients, we recover Lipschitz estimates and a (near-optimal) convergence rate. These results paves the way for studying complex structures of heterogeneous materials, *e.g.*, such as inclusions.

Références

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