Acyclic Gambling Games

Rida LARAKI, CNRS, Dauphine

Jérôme Renault, TSE

We consider 2-player zero-sum stochastic games where each player controls his own state variable living in a compact metric space. The terminology comes from gambling problems where the state of a player represents its wealth in a casino. Under standard assumptions (e.g. continuous running payoff and non expansive transitions), we consider for each discount factor the value v_λ of the λ-discounted stochastic game and investigate its limit when λ goes to 0. We show that under a new acyclicity condition, the limit exists and is characterized as the unique solution of a system of functional equations: the limit is the unique continuous excessive and depressive function such that each player, if his opponent does not move, can reach the zone when the current payoff is at least as good than the limit value, without degrading the limit value. The approach generalizes and provides a new viewpoint on the Mertens-Zamir system coming from the study of zero-sum repeated games with lack of information on both sides. A counterexample shows that under a slightly weaker notion of acyclicity, convergence of (v_λ) may fail.

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Rida LARAKI, CNRS, Lamsade, Université Paris Dauphine-PSL and University of Liverpool
rida.laraki@dauphine.fr