BD Entropy and Bernis Friedman Entropy

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We will propose a link between the BD entropy introduced by D.Bresch and B. Desjardins for viscous shallow water system in [1] and [2], and the dissipative Bernis Friedman entropy (denoted BF) introduced in [5] to study lubrication equations. Different dissipative entropies are obtained according to the choice of the drag term, generalizing by this some important work as in [7], [8], [6]. In the full generality, consider the following lubrication model

$$\partial_t h + \partial_x\left(\frac{1}{\alpha W_e} F(h)\partial_x^3 h - \frac{1}{\alpha Fr^2} D(h)\partial_x h\right) = 0. \tag{1}$$

Consider also a viscous shallow water model with a general representation of the surface tension and drag term

$$\begin{aligned} &\partial_t h_\epsilon + \partial_x(h_\epsilon u_\epsilon) = 0, \\ &\partial_t(h_\epsilon u_\epsilon) + \partial_x(h_\epsilon u_\epsilon^2) + \frac{1}{\epsilon Fr^2} S(h_\epsilon)\partial_x(h_\epsilon) = \frac{4}{R_e}\partial_x(h_\epsilon \partial_x u_\epsilon) + \frac{1}{\epsilon W_e} h_\epsilon \partial_x^3 h_\epsilon - \alpha \frac{h_\epsilon^2}{\epsilon T(h_\epsilon)} u_\epsilon. \end{aligned} \tag{2}$$

We will consider two cases of S and T, in which the link between the associated entropies (BD and BF) allows to obtain a weak solution of the lubrication model (1) from the weak solution of the shallow water model (2). It also allows to obtain some results on shallow water equations based on results obtained on the lubrication equations that have been much more studied historically [3], [4].

Références

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