Raviart-Thomas finite elements of Petrov-Galerkin type

François Dubois, CNAM et Université Paris-Sud

Isabelle GREFF, Université de Pau et des Pays de l'Adour

Charles Pierre, Université de Pau et des Pays de l'Adour

Finite volume methods are widely used, in particular because they allow an explicit and local computation of a discrete gradient. This computation is only based on the values of a given scalar field. In this contribution, we wish to achieve the same goal in a mixed finite element context of Petrov-Galerkin type so as to ensure a local computation of the gradient at the interfaces of the elements. The shape functions are the Raviart-Thomas finite elements, [2]. Our purpose is to define test functions that are in duality with these shape functions: precisely, the shape and test functions will be asked to satisfy some orthogonality property. This paradigm is addressed for the discrete solution of the Poisson problem. The general theory of Babuška brings necessary and sufficient stability conditions for a Petrov-Galerkin mixed problem to be convergent. In order to ensure stability, we propose specific constraints for the dual test functions. With this choice, we prove that the mixed Petrov-Galerkin scheme is identical to the four point finite volume scheme of Herbin, [1], and to the mass lumping approach developed by Baranger, Maitre and Oudin, [3]. Convergence is proven with the usual techniques of mixed finite elements.

Références

[1] R. Herbin. An error estimate for a finite volume scheme for a diffusion-convection problem on a triangular mesh. *Numer. Methods Partial Differential Equations*, 11(2):165–173, 1995.

[2] P.-A. Raviart and J.-M. Thomas. A mixed finite element method for 2nd order elliptic problems. *Mathematical aspects of finite element methods*, pages 292–315. Lecture Notes in Math., Vol. 606, 1977.

[3] J. Baranger, J.-F. Maitre, and F. Oudin. Connection between finite volume and mixed finite element methods. *RAIRO Modél. Math. Anal. Numér.*, 30(4):445–465, 1996.

François Dubois, Laboratoire de Mathématiques d'Orsay, Université Paris-Sud, CNRS, Université Paris-Saclay, 91405 Orsay, France.
`francois.dubois@u-psud.fr`
Isabelle GREFF, Laboratoire de Mathématiques et de leurs Applications de Pau, UMR CNRS 5142, Université de Pau et des Pays de l'Adour, France
`igreff@univ-pau.fr`
Charles Pierre, Laboratoire de Mathématiques et de leurs Applications de Pau, UMR CNRS 5142, Université de Pau et des Pays de l'Adour, France
`charles.pierre@univ-pau.fr`