# Numerical homogenization and the Arlequin method

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Our aim is to approximate a problem with highly oscillatory coefficients by a problem with coarse coefficients, when the oscillatory coefficients are not explicitly, or not entirely, known. Consider the problem

$$-\mathrm{div}(A_\varepsilon \nabla u_\varepsilon) = f,$$

where $A_\varepsilon(x)$ varies at the characteristic scale $\varepsilon$ (and may be random). Assume that this problem is to be solved for a large number of right-hand sides $f$. If the coefficient oscillations are infinitely rapid (i.e. $\varepsilon$ is asymptotically small), the solution $u_\varepsilon$ can be accurately approximated by the solution to the homogenized problem

$$-\mathrm{div}(A^\star \nabla u^\star) = f,$$

where the homogenized coefficient $A^\star$ has been evaluated beforehand by solving the corrector problem. If the oscillations are moderately rapid, one can think instead of MsFEM-type approaches to approximate the solution. However, in both cases, the complete knowledge of the oscillatory matrix coefficient $A_\varepsilon$ is required, either to build the average model (namely solve the corrector equation given by the homogenization theory and compute $A^\star$) or to compute the multiscale basis functions needed by the MsFEM approach. In many practical cases, this coefficient is only partially known, or merely completely unavailable, and one only has access to the solution $u_\varepsilon$ for any loading $f$ (e.g. using observations on actual experiments, or using a black-box numerical simulation code). Given these solutions $u_\varepsilon$, an interesting question is to find the best non-oscillating (or, more generally, slowly oscillating) matrix $\overline{A}$ (think e.g. of a constant matrix) that is consistent with the observed behavior. In a previous work [3], the last two authors have developed an approach to solve the problem by solving a minimization problem of the type

$$\inf_{\overline{A} \text{ constant matrix}} \ \sup_{f \text{ of unit norm}} \left\| u(A_\varepsilon, f) - u(\overline{A}, f) \right\|_{L^2},$$

where $u(A_\varepsilon, f)$ and $u(\overline{A}, f)$ respectively denote the solution of the diffusion problem with coefficient matrix $A_\varepsilon$ and $\overline{A}$, for the *same* right-hand side $f$. The work reported on in the present contribution is devoted to applying the same approach in the context of an Arlequin type method (see e.g. [2]): a portion of the considered computational domain is modeled using the original oscillatory matrix $A_\varepsilon$ and another portion is modeled using the matrix $\overline{A}$; the two portions being coupled using the Arlequin approach in order to reduce finite size and boundary effects. The approach has been pioneered in [1]. A mathematical formalization of the approach, its numerical analysis (proofs of consistency and convergence), along with various algorithmic improvements are now provided [4]. This work is partially supported by EOARD under Grant FA9550-17-1-0294.

## Références

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