Equations de Hamilton-Jacobi sur réseaux et schéma semi-Lagrangien

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The goal of this talk is to present and to justify a semi-Lagrangian scheme for an Hamilton-Jacobi equation posed on a network and modelling trafic flow problems (as studied in the work of Imbert and Monneau [3]). In a first part of the talk, we will explain how it is possible to deduce macroscopic models of traffic flow (using Hamilton-Jacobi formulation) from microscopic ones. The main advantage of microscopic models (in which we describe the dynamics of each vehicle in an individual way) is that one can easily distinguish each vehicle and then associate different attributes (like maximal velocity, maximal acceleration...) to each vehicle. It is also possible to describe microscopic phenomena like red lights, slowdown or change of the maximal velocity. The main drawback is for numerical simulations where we have to treat a large number of data, which can be very expensive for example if we want to simulate the traffic at the scale of a town.

On the contrary, macroscopic models consist in describing the collective behaviour of the vehicles for example by giving an evolution law on the density of vehicles. The oldest macroscopic model is the LWR model (Lighthill, Whitham [4], Richards [5]), which dates back to 1955 and is inspired by the laws of fluid dynamics. More recently, some macroscopic models propose to describe the flow of vehicles in terms of the averaged spacing between the vehicles. The main advantage of these macroscopic models is that it is possible to make numerical simulations on large portion of road. On the other side, it is more complicated to describe microscopic phenomena or attributes.

The first goal of this talk is to show how to pass from microscopic models to macroscopic ones. As we will explain, this problem can be seen as an homogenization result on a non-local Hamilton-Jacobi equation.

The second goal of this talk is to present a semi-Lagrangian scheme for the approximation of a class of Hamilton-Jacobi-Bellman equations on networks modelling macroscopic trafic flow problems. The scheme is explicit and stable under some technical conditions. We will prove a convergence result and some error estimates. Additionally, the theoretical results will be validated by numerical tests.

Références

[1] E. Carlini, A. Festa, N. Forcadel, *A semi-lagrangian scheme for hamilton-jacobi equations on networks*, Preprint.

[2] N. Forcadel, W. Salazar, M. Zaydan, *Specified homogenization of a discrete traffic model leading to an effective junction condition*, To appear in Communications on Pure and Applied Analysis..

[3] C. Imbert, R. Monneau, *Flux-limited solutions for quasi-convex hamilton-jacobi equations on networks*, Annales Scientifiques de l'ENS, 50 (2017), pp. 357–448.

[4] M. J. Lighthill, G. B. Whitham, *On kinematic waves. ii. a theory of traffic flow on long crowded roads* in Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, vol. 229, The Royal Society, 1955, pp. 317–345.

[5] P. I. Richards, *Shock waves on the highway*, Operations research, 4 (1956), pp. 42?51.

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