# Guided modes in a hexagonal periodic graph like domain : the zigzag and the armchair cases

**Sonia Fliss**, POEMS, UMR 7231 ENSTA-CNRS-INRIA

**Berangére Delourme**, LAGA, Université Paris 13

We consider the propagation of acoustic waves in a particular periodic medium which consists of the plane $\mathbb{R}^2$ minus an infinite set of equispaced hexagonal perfect conductor obstacles. The distance between each obstacle is supposed to be small. We introduce two types of unbounded lineic defects, the zigzag one and the armchair one, by changing the distance between two obstacles from each side of a line having respectively a zigzag form or an armchair one. Our aim is to find guided modes, that is to say solutions of the homogeneous wave equation propagating along the defect. It is well known that the guided modes are related to a spectral problem. We want, in this work, to exhibit conditions on the perturbations which ensure existence of guided modes.

Inspired by previous works on square lattices [1], we analyse the spectral problem by using a standard approach of asymptotic analysis. We first find the formal limit of the eigenvalue problem as the distance between the obstacles tends to 0. It corresponds to an eigenvalue problem for a second differential operator defined along an hexagonal graph. We proceed then to explicit computations of the spectrum. Finally we prove that the spectrum of the initial operator is close to the spectrum of the limit operator, as it is done in [2].

We show in particular that the conditions which ensure existence of guided modes are completely different if the perturbation follows a zigzag line and an armchair one. More precisely, the zigzag perturbations exhibit a certain stability (whose precise definition will be given) that the armchair perturbations have not. Similar results have been obtained for the Schroedinger operator in honeycomb structures in [3].

We will illustrate all the theoretical results by numerical simulations.

**Références**

[1] Delourme, B., Fliss, S., Joly, P., & Vasilevskaya, E., *Trapped modes in thin and infinite ladder like domains. Part 1: existence results.* Asymptotic Analysis, 103(3), 103-134, 2017.

[2] Post O., *Spectral convergence of quasi-one-dimensional spaces.* Ann. Henri Poincare, 7(5) :933 973, 2006

[3] Fefferman C.L. and Weinstein M.I., *Edge states of continuum Schroedinger operators for sharply terminated honeycomb structures*, arXiv preprint arXiv :1810.03497, 2018

**Sonia Fliss**, ENSTA PARISTECH 828, boulevard des Marchaux 91762 Palaiseau Cedex
sonia.fliss@ensta-paristech.fr