Stability and discretization of Feynman–Kac dynamics

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Sequential Monte Carlo is a family of algorithms widely used to estimate rare event probabilities. The method generally consists in running a set of replicas of a given system, and to select replicas depending on their position in the state space. With this procedure, one can enhance the number of replicas in regions that a single system would visit rarely. Observing the long time behavior of such algorithms provides information on the probabilities of rare events in a single system.

From a more general perspective, these algorithms rely on Feynman-Kac semigroups, that can be considered in continuous or discrete time. Compared to usual Markov chains, the trajectories of such system are weighted according to some function, which leads to a selection mechanism. In the context of rare events, it is important to understand the long time behavior of these dynamics and their discretizations, and to identify the leading error term induced by the numerical scheme when a time discretization is performed.

However, the stability properties depend both on the dynamics itself and on the weighting function, so the stability of the dynamics is dictated by an interplay between these two features. I will present a new approach for studying the stability of Feynman-Kac semigroups and their discretizations. It draws ideas from the theory of stochastic stability developed e.g. by Meyn and Tweedie, as well as from large deviations theory, which is a closely related field. The conditions we derive for the stability to hold have a natural interpretation, and we show that they apply to a number of interesting systems [1]. If time allows, I will present numerical analysis results to quantify the order of convergence of a numerical scheme to the continuous reference, and show relevant applications [2].

Références

[1] G. Ferré, M. Rousset and G. Stoltz, *More on the long time stability of Feynman–Kac semigroups*, 2018, arXiv:1807.00390.

[2] G. Ferré and G. Stoltz, *Error estimates on ergodic properties of Feynman–Kac semigroups*, 2018, arXiv:1712.04013.

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