A discrete time Dynamic Programming approach on a tree structure for finite horizon optimal control problems

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The classical Dynamic Programming (DP) approach to optimal control problems is based on the characterization of the value function as the unique viscosity solution of a Hamilton-Jacobi-Bellman (HJB) equation [2]. The DP scheme for the numerical approximation of viscosity solutions of those equations is typically based on a time discretization which is projected on a fixed space triangulation of the numerical domain [3]. The time discretization can be done by a one-step scheme for the dynamics and the projection on the grid typically uses a polynomial interpolation. This approach, which allows to get information on optimal controls in feedback form, has been shown to be very powerful for low dimensional optimal control problems although general convergence results have been proved in $\mathbb{R}^d$. Several methods have been proposed to mitigate the curse of dimensionality of DP schemes, e.g. static and dynamic domain decomposition, fast-marching and fast-sweeping methods, discrete representation formulas (when available), see [3] and the references therein.
We will discuss a new approach for finite horizon optimal control problems [1, 4] where we compute the value function on a tree structure generated by the time discrete dynamics avoiding the construction of a space grid/triangulation to solve the HJB equation. This allows to drop the cost of the space interpolation, moreover the tree will guarantee a perfect matching with the discrete dynamics. In the simple case, we discretize the dynamics with an Euler scheme and we will prove first order convergence by the means of viscosity solutions. We will also discuss how this approach can be extended to high-order schemes and we will show some examples of second order approximation schemes.

Références

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