# The Gradient Discretisation Method to analyse numerical schemes for the elliptic part of a dispersive shallow water system

**Virgile DUBOS**, Sorbonne Université

**Cindy GUICHARD**, Sorbonne Université

**Yohan PENEL**, INRIA Paris

**Jacques SAINTE-MARIE**, INRIA Paris

**Mots-clés :** shallow water flows, dispersive effects, gradient discretisation method

We propose to use the Gradient Discretisation Method (GDM) [1] to analyse numerical schemes for the elliptic part of a family of dispersive Shallow Water models.
The considered family has been introduced in [2], it depends on a parameter and contains the well-known Green-Naghdi model. These models are derived from the incompressible Euler system with free surface and without the hydrostatic assumption. Hence, the studied models appear as extensions of the classic shallow water system enriched with dispersive terms. The model formulation motivates to use a prediction-correction scheme for its numerical approximation (see [3]). The prediction part leads to solving a shallow water system with topography, while the correction part leads to solving an elliptic-type problem.
The GDM is a framework which contains classic discretisation schemes for diffusion problems of different kinds: linear or non-linear, steady-state or time-dependent. The schemes may be conforming or non-conforming, low or high order, and may be built on very general meshes. This framework is extended with the Abstract Gradient Discretisation Method (AGDM) which allows to deal with non classic operators and various boundary conditions more simply. We present the core properties required to prove the convergence of GDM and AGDM, then, both methods are applied on the elliptic part of the studied problem and we discuss the results.

### Références

[1] Jérôme Droniou, Robert Eymard, Thierry Gallouët, Cindy Guichard, Raphaèle Herbin, *The gradient discretisation method*, Springer International Publishing AG, 2018.

[2] Marie-Odile Bristeau, Anne Mangeney, Jacques Sainte-Marie, Nicolas Seguin, *An energy-consistent depth-averaged Euler system: derivation and properties*, Discrete and Continuous Dynamical Systems - Series B, American Institute of Mathematical Sciences, 2015.

[3] Nora Aissiouene, Marie-Odile Bristeau, Edwige Godlewski, Jacques Sainte-Marie, *A combined finite volume - finite element scheme for a dispersive shallow water system*, Networks and Heterogeneous Media, AIMS-American Institute of Mathematical Sciences, 2016.

**Virgile DUBOS**, LJLL, Sorbonne Université, 4 place Jussieu, 75005 Paris
dubos@ljll.math.upmc.fr
**Cindy GUICHARD**, LJLL, Sorbonne Université, 4 place Jussieu, 75005 Paris
cindy.guichard@sorbonne-universite.fr
**Yohan PENEL**, ANGE, INRIA Paris, 2 rue Simone Iff, 75012 Paris
yohan.penel@inria.fr
**Jacques SAINTE-MARIE**, ANGE, INRIA Paris, 2 rue Simone Iff, 75012 Paris
jacques.sainte-marie@inria.fr