Adaptive mesh refinement strategies on structured meshes for the mixed finite element discretization of the neutron diffusion equation

Minh-Hieu DO, CEA-Saclay

Patrick CIARLET, ENSTA-Paristech

François MADIOT, CEA-Saclay

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The neutron diffusion equation can be used to model the physics of the nuclear reactor core. The solution of this equation depends on several variables which consists of the neutron position in space, the motion direction, the neutron energy and it corresponds to a generalized eigenvalue problem defined as

Find $(\mathbf{p}, \phi, \mathrm{k_{eff}}) \in \mathbf{H}(\mathrm{div}, \mathcal{R}) \times \mathrm{H}^1_0(\mathcal{R}) \times \mathbb{R}^{+*}$ such that:

$$\left\{ \begin{array}{rcll} \mathcal{D}^{-1}\,\mathbf{p} + \mathbf{grad}\,\phi & = & 0, & \text{in } \mathcal{R}, \\ \mathrm{div}\,\mathbf{p} + \Sigma_\mathrm{a}\,\phi & = & \frac{1}{\mathrm{k_{eff}}}\Sigma_f\phi, & \text{in } \mathcal{R}, \\ \phi|_{\partial\mathcal{R}} & = & 0, & \text{on } \partial\mathcal{R}, \end{array} \right.$$

where $\mathcal{R}$ is regular open bounded domain, $\mathbf{p}$ represents the neutron current and ϕ stands for the neutron scalar flux. The absorption cross section (Σ_a), the fission cross section (Σ_f) and diffusion coefficient ($\mathcal{D}$) are assumed piecewise constant. The unknown $\mathrm{k_{eff}}$ is the inverse of the eigenvalue and its value characterizes the physical state of the core reactor. According to the Krein-Rutman theorem [1], the only physical solution is the eigenfunction associated to the smallest eigenvalue (the largest $\mathrm{k_{eff}}$). In order to compute this solution, we often use the inverse power iteration with solving a source problem at each iteration.

Classically, this problem can be recast in a mixed variational form, and then discretized with the help of the Raviart-Thomas-Nédélec Finite Element. Since the macroscopic cross section are piecewise constant, it gives low regularity solutions for the neutron diffusion equation [2]. Hence, it is important to refine the mesh at the singular regions to gain a better estimate for the solution. One of the most popular way to treat this problem is the Adaptive Mesh Refinement (AMR)[3]. In this work, we study a posteriori estimate and some marker cell strategies for the mesh refinement for cartesian meshes. Moreover, we also propose an algorithm which combines the AMR process with the inverse power iteration to handle the low regularity problem.

References

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Minh-Hieu DO, CEA/DEN/DM2S/SERMA/LLPR, CEA - Saclay, 91191 Saclay
minh-hieu.do@cea.fr
Patrick CIARLET, Poems, UMA, ENSTA Paristech, 828 Boulevard des Maréchaux, 91120 Palaiseau
patrick.ciarlet@ensta-paristech.fr
François MADIOT, CEA/DEN/DM2S/SERMA/LTSD, CEA - Saclay, 91191 Saclay
francois.madiot@cea.fr