A Pontryagin maximum principle for optimal sampled-data control problems with free sampling times and running state constraints

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Established in the 1950s by Pontryagin, Boltyanskii, Gamkrelidze and Mishchenko, the Pontryagin maximum principle (in short, PMP) is a fundamental result in optimal control theory. It gives a first-order necessary condition for the optimal control among all *permanent* controls, that is to say when the value of the control can be changed at any moment in time. Since then the PMP has been adapted to many control systems of different natures. Notably recent works are interested in *sampled-data control* systems. A sampled-data control system is a dynamical system in which the state evolves continuously with respect to time, while the control can only be modified a finite number of times. Recently Bourdin and Trélat have obtained in [1] a version of the PMP for optimal sampled-data control problems with fixed sampling times.

The work [1] does not take into account the possibility of *free sampling times*, where one is allowed to optimize the times when the control can be modified. In this talk we will present a new version of the PMP for optimal sampled-data control problems with free sampling times. We find that the additional necessary condition for the optimal sampling times coincides with the continuity of the Hamiltonian function (see [2]). Recall that this property is a well-known fact for optimal permanent controls, while it is not preserved in general for optimal sampled-data controls with fixed sampling times. Therefore, our result asserts that the continuity property is recovered in the case of optimal sampled-data controls with optimal sampling times and moreover it can be used, for example in shooting methods, to determine the optimal sampling times.

Finally we will also discuss a new version of the PMP in the case of optimal sampled-data control problems with running state constraints (see [3]). While the statement of the PMP for optimal permanent control problems is more involved since the adjoint vector is now described as a function of bounded variations, we are able, in the case of sampled-data control, to provide general sufficient conditions ensuring that the singular part of the adjoint vector is zero and, above all, ensuring that the jumps of the adjoint vector occur only at the sampling times.

Références

[1] L. Bourdin, E. Trélat, *Optimal sampled-data control, and generalizations on time scales*, Mathematical Control and Related Fields, 2016.

[2] L. Bourdin, G. Dhar, *Continuity/constancy of the Hamiltonian function in a Pontryagin maximum principle for optimal sampled-data control problems with free sampling times*, submitted for publication.

[3] L. Bourdin, G. Dhar, *Optimal sampled-data control problems with running state constraints Pontryagin maximum principle and bouncing trajectories phenomenon*, work in progress.

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