Boundary null-controllability of one-dimensional coupled parabolic system with Fourier conditions

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This work is concerned with the boundary null-controllability property of some 2×2 linear coupled parabolic system with Fourier (Robin) boundary conditions. More precisely, two one-dimensional parabolic equations are considered with a linear coupling of first component inserted through the second equation and here we exert a single control input at one boundary point. We take our system as follows:

$$\begin{cases} \partial_t y_1 - \partial_x(\gamma(x)\partial_x)y_1 = 0 & \text{in } (0,T)\times(0,1), \\ \partial_t y_2 - \partial_x(\gamma(x)\partial_x)y_2 + y_1 = 0 & \text{in } (0,T)\times(0,1), \\ \gamma(x)\frac{\partial y_1}{\partial \nu}(t,x) + \beta_1 y_1(t,x) = 1_{\{x=0\}}v(t) & \text{on } (0,T)\times\{0,1\}, \\ \gamma(x)\frac{\partial y_2}{\partial \nu}(t,x) + \beta_2 y_2(t,x) = 0 & \text{on } (0,T)\times\{0,1\}, \\ y_1(0,x) = y_{01}(x) & \text{in } (0,1), \\ y_2(0,x) = y_{02}(x) & \text{in } (0,1), \end{cases}$$

where we assume the diffusion coefficient $\gamma \in C^1([0,1];\mathbb{R})$ with $\gamma_{\min} := \inf_{[0,1]} \gamma > 0$, β_1 & β_2 are two positive real parameters, $1_{\{x=0\}}$ is the characteristic function of the point $x = 0$, $y_{01}, y_{02} \in L^2(0,1)$ and the control input v is supposed to be in $L^2(0,T)$.

For the above problem, we prove the existence of a null-control $v \in L^2(0,T)$ satisfying the bound

$$\|v\|_{L^2(0,T)} \leq C_{T,\gamma}(1+\beta_1)\left(\|y_{01}\|_{L^2(0,1)} + \|y_{02}\|_{L^2(0,1)}\right)$$

under the following cases:

case (i) when $0 < \beta_1 = \beta_2 (=: \beta) < +\infty$ and any γ as introduced above and
case (ii) when $\beta_1, \beta_2 \in (0, +\infty)$, $\beta_1 \neq \beta_2$ with $\gamma \equiv 1$.

The proof follows from the method of moments which needs some suitable spectral properties of the *adjoint* to the corresponding elliptic operator of our parabolic system. But the spectral analysis changes from case (i) to case (ii) and this is the reason to split our problem into two different cases.

We also prove for both cases that as the parameters β_1, β_2 goes to 0 or, $+\infty$ (at least up to a subsequence), the corresponding solution converge to a solution of *Neumann* or, *Dirichlet* boundary control problem. The later is important in the sense of a *penalization approach* to solve a non-homogeneous Dirichlet boundary problem.

Références

[1] A. Benabdallah, F. Boyer, and M. Morancey, *A block moment method to handle spectral condensation phenomenon in parabolic control problems*, preprint, https://hal.archives-ouvertes.fr/hal-01949391, Dec. 2018.

[2] K. Bhandari and F. Boyer, *Boundary null-controllability of one-dimensional coupled parabolic system with Fourier conditions*, In preparation, 2019.

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