Multiscale Finite Element Method for thin domains

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Advances in engineering make composite plates an interesting subject of study for industry. Mathematically their behaviour is typically modelled by elliptic partial differential equations with highly oscillating coefficients on a domain which has a dimension much smaller than the others ([1], [2]). Numerical difficulties arise because of the different length scales involved, which lead to prohibitive computation costs if standard (say Finite Element) approaches are used. To solve this problem we turn to methods usually used in homogenization theory.
To address such difficult cases in the case of three dimensional elasticity, several numerical multiscale approaches have been proposed in the literature, including the Multiscale Finite Element Method (MsFEM) ([3]). In our work, we show how to adapt the MsFEM to the case of plates, in the regime when the typical small size of the microstructure is of the order of the thickness of the plate. Such an approach allows to compute an accurate approximation of the 3D oscillatory solution on a 2D coarse mesh by using dedicated basis functions. Very significant gains in terms of computation costs are therefore obtained.

Références

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