On hyperbolic systems with stiff source terms

T. Pichard, N. Aguillon, B. Després, E. Godlewski

This work focuses on physical flows with threshold effects modeled by hyperbolic systems of the form

$$\partial_t U + \partial_x F(U) = S(U), \tag{1}$$

where the source S is a discontinuous function of the unknown. As an application, we consider a homogenized two-phase model described by

$$U = (\alpha\rho_v,\ \rho,\ \rho u,\ \rho E), \qquad F(U) = \left(\alpha\rho_v u,\ \rho u,\ \rho u^2 + p,\ (\rho E + p)u\right),$$
$$S(U) = \left(K\phi\mathbf{1}_{h(U)>h^{eb}},\ 0,\ 0,\ \phi\right),$$

where $\alpha\rho_v$ is the density of vapor, ρ is the density of homogenized fluid, u its velocity, E its total energy and h its enthalpy. The source S models the heating of the fluid and the change from the liquid phase to the vapor phase at enthalpy $h = h^{eb}$.

In equations of the form (1), such a source term S discontinuous in the unknown lead to difficulties at the numerical level and naive approaches commonly lead to non-physical wave speeds and oscillations ([1]).

In a first step, we focus on the existence and uniqueness of stationary states for (1). Due to the discontinuity of the source S, Cauchy-Lipschitz theorem holds not here, and need to be generalized.

In a second step, we aim to construct well-balanced schemes (see *e.g.* [2]) for (1) preserving the stationary states and without non-physical numerical artefact.

References

[1] R. J. Leveque and H. C. Yee. A study on numerical methods for hyperbolic conservation laws with stiff source term, NASA report, 1988

[2] A. Bermudez and M. E. Vazquez. Upwind methods for hyperbolic conservation laws with source terms, Comput. & Fluids, 23(8), pp. 1049-1071, 1994