Gradient Exact Enlarged Controllability (G.E.E.C) of the semilinear heat equation

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In this work we study the gradient enlarged exact controllability of the semilinear heat equation just in an intern subregion ω of the system evolution domain Ω.
Thus, let Ω be an open bounded set of $\mathbb{R}^n (n \geq 1)$ with regular boundary $\partial\Omega$. We consider the Banach space $H = L^2(\Omega)$, and the corresponding norm $\|.\|_H$. For a given $T > 0$, we denote $Q_T = \Omega \times]0, T[$, $\Sigma_T = \partial\Omega \times]0, T[$ and let us consider the following problem :

$$\text{Minimize} \quad J(u) = \frac{\alpha}{2} \|\chi_\omega \nabla y_u(T) - y_d\|^2_{L^2(\omega)} + \frac{\beta}{2} \|u\|^2_{L^2(\Omega)} \tag{1}$$

$$\text{subject to:} \begin{cases} \partial_t y(x,t) - \mathcal{A}y(x,t) - \mathcal{N}y(x,t) = Bu(t) & Q_T \\ y(x,0) = y_0(x) & \Omega \\ y(\xi,t) = 0 & \Sigma_T \ , \end{cases} \tag{2}$$

The problem (1-2) is well-posed and has a unique solution [1]. To compute the control u we use the Lagrangian multiplier approach and for the numerical approach we use Uzawa algorithm [2, 3, 4].

Références

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