Energy-based operator splitting approach for the time discretization of nonlinear ordinary differential equations coupled with Stokes systems

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Multiscale coupling of nonlinear lumped and distributes fluid flow models is of interest when modeling complex hydraulic networks *e.g.* oil ducts, water supply, biological flows, microfluidic channels *etc.* In the present work, we are motivated by the computational modeling of blood flow through the cardiovascular system and we focus on the multiscale coupling in terms of spatial dimensions, leading to the coupling between partial and ordinary differential systems.

We consider non-stationary Stokes systems modeling flow of an incompressible viscous fluid in rigid domains, coupled with nonlinear systems of ordinary differential equations (ODEs) representing lumped descriptions of the the flow in different parts of a complex hydraulic network. Multiple connections among Stokes domains and lumped circuits are allowed. At the coupling interface we impose continuity of pressure, via a natural Neumann condition, and continuity of flow [2].

First, we carefully derive an energy identity satisfied by the full coupled problem, that embodies the main mechanisms governing the physics of the system. Based on these considerations, we next develop a novel technique based on operator splitting for the time discretization of the problem that allows to solve separately and sequentially the Stokes problem and the ODEs without the need of sub-iterations [1]. The energy of the semi-discrete problem mirrors the behavior of the energy of the full coupled system, thereby providing unconditional stability to the proposed splitting method. Finally, to illustrate the performances of the proposed method, we examine several examples in which we solve in separate blocks nonlinearities coming from different sources. In particular, we derive explicit solutions for the full coupled problem in two different meaningful configurations, against which our (as well as other) numerical methods can be tested. The scheme presented yields, at most, a first-order accuracy in time, since it includes only two sub-steps. Further extensions and improvements, including variants that provide increased accuracy in time or ability to handle Navier-Stokes equations and fluid-structure interactions can be obtained by combining the proposed scheme with other operator splitting techniques already developed [3, 4].

Références

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