Null controllability of a linearized Korteweg-de Vries equation by backstepping approach

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This talk is based on the new research paper [5] and the references therein.
We consider the null controllability of the following linearized KdV control system:

$$\begin{cases} u_t + u_{xxx} + u_x = 0 & \text{for } (t,x) \in (s,+\infty) \times (0,L), \\ u(t,L) = u_x(t,L) = 0 & \text{for } t \in (s,+\infty), \\ u(t,0) = \kappa(t) & \text{for } t \in (s,+\infty), \end{cases} \tag{1}$$

where $\kappa(t) \in \mathbb{R}$ is the control term, which is a little different from the pioneer one raised by Lionel Rosier [3], the only difference is the place where control acts: $u(t,0)$ instead of $u_x(t,L)$. For the model given by Lionel Rosier, one surprisingly finds that the controllability of system depends on the length of the interval, more precisely the system is controllable if and only if

$$L \notin \mathcal{N} := \left\{ 2\pi \sqrt{\frac{l^2 + lk + k^2}{3}};\, l,k \in \mathbb{N}^* \right\}. \tag{2}$$

In the case of system (1), there is no critical length (but some regularity difficulties occur). The controllability was first discovered by Lionel Rosier in [4]. In [1], Jean-Michel Coron and Eduardo Cerpa have given the rapid stabilization for this system by using the backstepping method. Recently, in [2], using the backstepping approach Jean-Michel Coron and Hoai-Minh Nguyen proved the null controllability and the semi-global finite time stabilization for a class of heat equations. This turned out to be a method to reach possibly the semi-global finite time stabilization for those systems which can be rapidly stabilized by means of backstepping methods. At the same time, this provides a visible way to get the null controllability instead of using Hilbert Uniqueness Method by which one doesn't know explicitly what the control is.
In this talk, we show how this new method developped by Jean-Michel Coron and Hoai-Minh Nguyen can be used to prove the null controllability of system (1).

Theorem 1. *For any given $T > 0$, the controlled system (1) is null controllable in time T.*

Instead of using Carleman estimate in [4], we gave another prove of the null controllability of linearized KdV controlled system (1). Let us recall that the exact controllability of system (1) fails, which is also proved in [4].

Références

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[3] Lionel Rosier. Exact boundary controllability for the Korteweg-de Vries equation on a bounded domain. *ESAIM Control Optim. Calc. Var.*, 2:33–55 (electronic), 1997.

[4] Lionel Rosier. Control of the surface of a fluid by a wavemaker. *ESAIM Control Optim. Calc. Var.*, 10(3):346–380 (electronic), 2004.

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