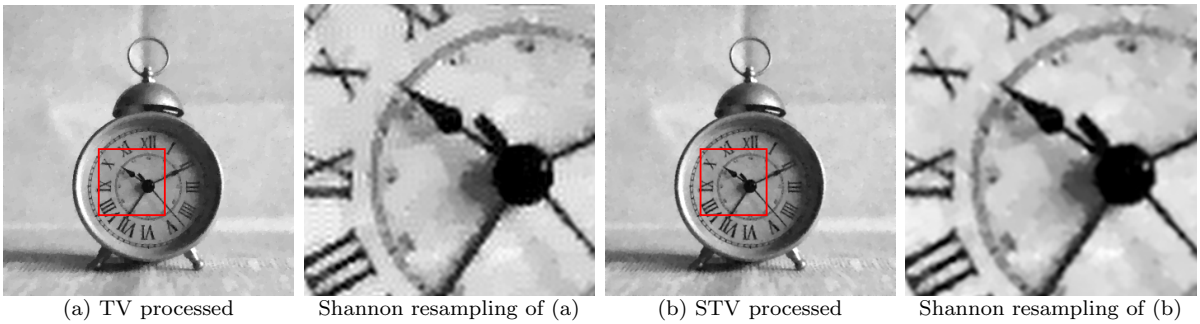
The Shannon Total Variation

Rémy ABERGEL, LTCI, Télécom ParisTech

Lionel MOISAN, Université Paris Descartes

Mots-clés : total variation, image interpolation, Shannon theory, image restoration, aliasing reduction.

In image processing problems, the minimization of total variation (TV) based energies requires discretization schemes, such as the commonly used finite differences approach. Unfortunately, such schemes generally lead to images which are difficult to interpolate (see Figure 1), which strongly restrict their potential use in image analysis algorithms requiring sub-pixel precision. This issue can be avoided by using the "Shannon" total variation (STV), a discretization that is inherently compatible with Shannon interpolation. We will explain how the STV regularization can be efficiently handled with modern primal-dual algorithms, and present several applications (image denoising, deblurring, spectrum extrapolation) where the improved behavior of the Shannon total variation yields images with better sub-pixel accuracy. We will also present a new STV-regularized optimization problem involving a data-fidelity term formulated in the frequency domain, which can be used to remove aliasing from an image, or given an image which is difficult to interpolate, can produce a visually similar image which can be easily interpolated.



(a) TV processed — Shannon resampling of (a) — (b) STV processed — Shannon resampling of (b)

Figure 1: **Manipulating TV or STV processed images at the subpixellic scale.** A noisy image (not represented here) corrupted by an additive Gaussian noise with zero mean and standard deviation $\sigma = 20$ was denoised by computing a TV proximal operator, image (a), or a STV proximal operator, image (b). Although both models yields visually similar images, zooming the TV processed image (a) using the Shannon interpolation reveals undesirables oscillations localized near object contours. This is not the case with the STV processed image (b) which can be interpolated without artifacts.

Références

[1] R. Abergel, L. Moisan, *The Shannon Total Variation*, preprint MAP5, 2016.

[2] L. Moisan, *How to discretize the total variation of an image?*, Proceedings in Applied Mathematics and Mechanic, 2007.

[3] F. Malgouyres and F. Guichard, *Edge direction preserving image zooming: a mathematical and numerical analysis*, SIAM Journal on Numerical Analysis, 2001.

[4] L. I. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Physica D: Nonlinear Phenomena, 1992.

Rémy ABERGEL, Laboratoire LTCI, Télécom ParisTech, 46 rue Barrault, 75013 Paris.
Remy.Abergel@telecom-paristech.fr
Lionel MOISAN, Laboratoire MAP5, 45 rue des Saints-Pères, 75006 Paris.
Lionel.Moisan@parisdescartes.fr