Mathematical properties of an integro-differential model from population genetics

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In this talk I will discuss a mathematical analysis of an integro-differential model arising in population genetics. The model describes the dynamics of fitness distribution in an asexual population under the effect of mutation and selection. These two processes are represented by two nonlocal terms. First, we prove the existence and uniqueness of the solution, and we derive asymptotic estimates of the distribution as the fitness tends to $\pm\infty$. Based on these asymptotic estimates, we then show that the cumulant generating function of the distribution is well-defined and satisfies a linear nonlocal transport equation that we solve explicitly. This explicit formula allows us to characterize the dependence of the long time behavior of the distribution with respect to the mutation kernel. On the one hand, if the kernel contains some beneficial mutations, the distribution diverges, which is reminiscent of the results of [1] who analysed a mutator-replicator equation with a diffusive mutation term. On the other hand, if the initial fitness distribution admits some upper bound, purely deleterious kernels lead to the convergence of the distribution towards an equilibrium. The shape of the equilibrium distribution strongly depends on the kernel through its harmonic mean $-s_H$: the distribution admits a positive mass at the best initial fitness class if and only if $s_H \neq 0$.
The talk is based on a joint work with Franois Hamel (I2M, AMU), Guillaume Martin (ISEM, CNRS) and Lionel Roques (BioSP, INRA).

Références

[1] Alfaro, M and Carles, R, *Explicit Solutions for Replicator-Mutator Equations: Extinction versus Acceleration*, SIAM Journal on Applied Mathematics, 2014.

[2] Martin, G and Roques, L, *The Non-stationary Dynamics of Fitness Distributions: Asexual Model with Epistasis and Standing Variation*, Genetics, 2016.

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