Positive lower bound for the numerical solution of a convection-diffusion equation

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In a famous paper of 1957, Ennio De Giorgi [1] (see also [2]) introduced a very robust method to establish local L^∞-bounds and local Hölder regularity for the solutions of elliptic equations with rough coefficients. We adapt this method in a discrete setting. More precisely, given a bounded polyhedral domain $\Omega \subset \mathbf{R}^N$ and a source $f \in L^1(\Omega)$, we consider the following system

$$-\Delta\Psi = f \qquad \text{in } \Omega, \tag{1}$$

$$\operatorname{div}(-\nabla v + \nabla\Psi v) = 0 \qquad \text{in } \Omega. \tag{2}$$

This system is supplemented with mixed Dirichlet-Neumann boundary conditions. We assume that $\partial\Omega = \Gamma^D \cup \Gamma^N$ with $\Gamma^D \cap \Gamma^N = \emptyset$ and $\mathrm{m}(\Gamma^D) > 0$ and we consider $v^D \in L^\infty(\Gamma^D)$, $\Psi^D \in L^\infty(\Gamma^D)$. The boundary conditions write:

$$\Psi = \Psi^D,\ v = v_D \quad \text{on } \Gamma^D \qquad \text{and} \qquad \partial_n\Psi = \partial_n v = 0 \quad \text{on } \Gamma^N. \tag{3}$$

We consider the discretization of some stationary convection-diffusion equation by any stable Finite Volume method based on a two point flux approximation. We show that if $v_D \geq 1$, then the discrete solution v_h is positive in Ω, with a positive lower bound which does not depend on the size of the mesh.

Références

[1] De Giorgi, Ennio, *Sulla differenziabilità e l'analiticità delle estremali degli integrali multipli regolari.*, Mem. Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. (3) 3, 25–43 (1957).

[2] Vasseur, Alexis, *The De Giorgi method for elliptic and parabolic equations and some applications, Part 4*, Morningside Lect. Math., vol. 4, pp. 195–222. Int. Press, Somerville, MA (2016).

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