Pointwise convergence of the hP discontinuous Galerkin finite element method for elliptic problems with point singularities

Carlo MARCATI, LJLL, UPMC Paris 6

Yvon MADAY, LJLL, UPMC Paris 6

Many problems arising from science and engineering have solutions that are regular everywhere except for a finite set of points. This is the case, for example, of elliptic equations in domains with corners, where the loss of regularity is due to the shape of the domain, and of the electronic Schrödinger operator $-\Delta + V$ in the case of a molecule, where V is a potential with singularities at the positions of the nuclei. Through geometrical mesh grading and a combination of low- and high-order polynomial approximation – see Figure 1 for an illustration – the hP finite element method (FEM) provides numerical solutions that, under some conditions, converge *exponentially* to the exact solution. We apply the hP FEM in the context of a discontinuous Galerkin (dG) approximation, which provides a great flexibility from both the theoretical and computational points of view.
In this talk we show that given a solution u of an elliptic problem with point singularities and an hP dG approximation obtained with N degrees of freedom, there exist $C, b > 0$ such that

$$\|u - u_\delta\|_{L^\infty(\Omega)} + \|r^{1-\gamma}\nabla(u - u_\delta)\|_{L^\infty(\Omega)} \leq C \exp(-bN^{1/(d+1)})$$

for a sufficiently small $\gamma > 0$, in a domain $\Omega \subset \mathbb{R}^d$, and where r represents the distance from the singularity. In the analysis, we also give some bounds on the Green functions of the operators under consideration, which can be of independent interest.

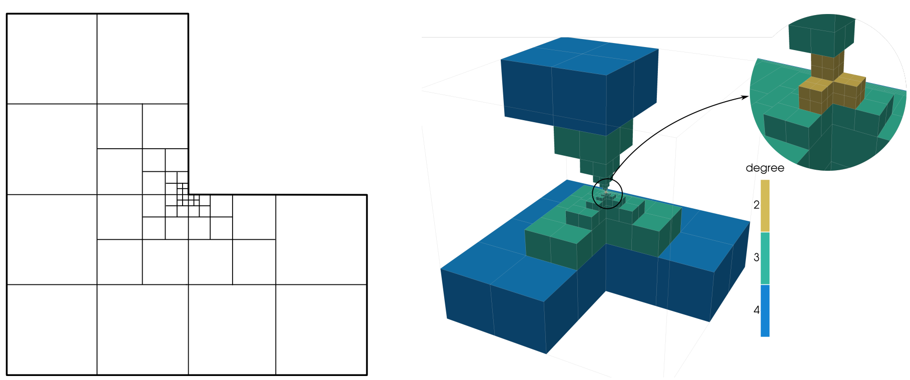

Figure 1: Two and three dimensional meshes.

Carlo MARCATI, Laboratoire Jacques Louis Lions, Université Pierre et Marie Curie, 4 place Jussieu, 75005 Paris
`carlo.marcati@upmc.fr`
Yvon MADAY, Laboratoire Jacques Louis Lions, Université Pierre et Marie Curie, 4 place Jussieu, 75005 Paris
`yvon.maday@upmc.fr`