Study of an isothermal phase transition by traveling waves

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Key words : systems of conservation laws, traveling waves, phase transition.

We consider a compressible two-phase medium undergoing isothermal transformations whose specific volume and velocity are respectively τ and u. The phase $k = liq, vap$ of the medium is equipped with an Equation of State (EOS) given as the pressure law $P_k \mapsto \tau_k(P_k)$, where τ_k and P_k are the specific volume and the pressure of phase k. The composition of the medium is characterized by y that denotes the mass fraction of the vapor phase and that verifies $\tau = y\tau_{\text{vap}} + (1-y)\tau_{\text{liq}}$. We assume a continuous pressure equilibrium between both phases in the system, namely $P_{\text{vap}} = P_{\text{liq}} = P$.

Let $P_k \mapsto g_k$ denotes the chemical potential of phase $k = \text{liq}, \text{vap}$. Under classic hypotheses, there exists P^* such that $g_{\text{vap}}(P^*) = g_{\text{liq}}(P^*)$. Let us note $\tau_k^* = \tau_k(P^*)$, $k = \text{vap}, \text{liq}$. We suppose that our two-phase medium can undergo mass transfer phenomena that reach an equilibrium characterized by $(P, y) = (P_{\text{eq}}, y_{\text{eq}})(\tau)$. The definition of P_{eq} and y_{eq} give the mass fraction and the pressure law for a given τ at equilibrium with respect to phase change effects.

$$(P_{\text{eq}}, y_{\text{eq}})(\tau) = \begin{cases} (P_{\text{liq}}(\tau), 0), & \text{if } \tau < \tau^*_{\text{liq}}, \\ \left(P^*, \frac{\tau - \tau^*_{\text{liq}}}{\tau^*_{\text{vap}} - \tau^*_{\text{liq}}}\right), & \text{if } \tau^*_{\text{liq}} \le \tau \le \tau^*_{\text{vap}}, \\ (P_{\text{vap}}(\tau), 1), & \text{if } \tau > \tau^*_{\text{vap}}. \end{cases}$$

We now suppose that the flows in the medium are governed by the following extended p-system :

$$\begin{cases} \partial_t \tau - \partial_x u = 0, \\ \partial_t u + \partial_x P = 0, \\ \partial_t y = (P_{\text{eq}}(\tau) - P)/\varepsilon, \end{cases} \tag{1}$$

that involves relaxation effects induced by phase change.

Formally, when $\varepsilon \to 0$ we recover the classic p-system closed by the pressure law $\tau \mapsto P_{\text{eq}}$. In our study, we try to understand how the solutions of the system (1) tend to those of the p-system when $\varepsilon \to 0$. We perform an analysis of traveling wave solutions for system (1). This will allow to define admissible discontinuities of the classic p-system by selecting discontinuities that match the relaxation limit $\varepsilon \to 0$. This method is used in many works[1, 2, 3]. One of the difficulties is that the term of relaxation can degenerate, leading to a loss of hyperbolicity when $\tau \in (\tau^*_{\text{liq}}, \tau^*_{\text{vap}})$. Nevertheless, we recover that the admissible discontinuities satisfy the Liu criterion [4].

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