Large Spectral Partitions

Beniamin BOGOSEL, École Normale Supérieure, Paris

Bourdin, Bucur and Oudet introduced in [1] an algorithm for the numerical study of partitions $(\omega_i),\ i = 1, ..., n$ of a domain D in $\mathbb{R}^2$ which minimize the quantity

$$\lambda_1(\omega_1) + \lambda_1(\omega_2) + ... + \lambda_1(\omega_n).$$

Their idea was to replace each set by a density function and use a relaxed formulation in order to compute approximations of the eigenvalues. For each cell ω_i if we consider an approximation φ of its characteristic function, then the problem

$$-\Delta u + C(1-\varphi)u = \lambda u,\ u \in H^1(D),\ C \gg 1$$

gives us an approximation of the eigenvalues. In the above algorithm, the authors solve this problem by using a discretization on a fixed computational grid of the size of D. While this simplifies the treatment of the partition condition, it turns out that for large number of cells the method is computationally expensive.

We propose an amelioration of the above algorithm. While still working on a fixed grid, it turns out that it is not necessary to use all the points in order to compute the value of our functional. Indeed, if we consider a restricted sub-grid around each cell, the precision is increased, while the computational cost decreases significantly. This allows us to study large spectral partitions with low resources (on a personal computer). Moreover, the algorithm can be applied to the study of partitions of three dimensional surfaces or even volumic partitions in 3D.

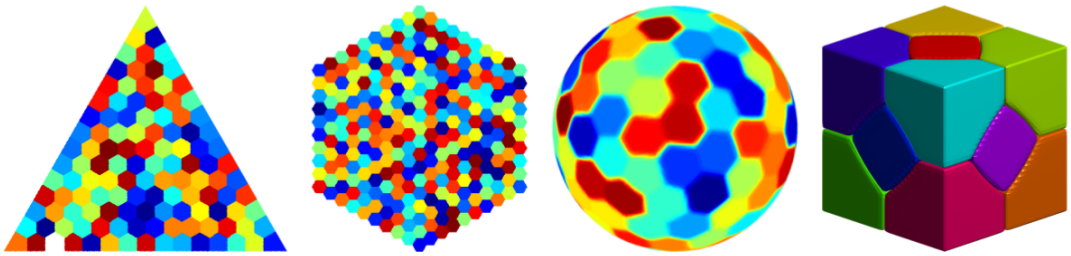

Figure 1: Numerical results: plane domains, surfaces, 3D domains

Références

[1] B. Bourdin, D. Bucur, and É. Oudet. Optimal partitions for eigenvalues. *SIAM J. Sci. Comput.*, 31(6):4100–4114, 2009/10.

Beniamin BOGOSEL, DMA - École Normale Supérieure 45, rue d'Ulm 75230 Paris Cedex 05 - France
beniamin.bogosel@ens.fr