A Technique For Choosing the Weights for Weighted ℓ_1 -minimization.

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A prominent approach to the problem of recovering a sparse vector $x_0 \in \mathbb{R}^d$ (meaning that only few coefficients $x_0(i)$ are non-vanishing) from underdetermined linear measurements $b = Ax_0$ is ℓ_1 -minimization:

$$\min \|x\|_1 \text{ subject to } Ax = b. \tag{\mathcal{P}_1}$$

For many different types of measurement matrices $A \in \mathbb{R}^{m,d}$, it can be shown that the solution to (\mathcal{P}_1) is equal to an *s*-sparse ground truth signal x_0 with high probability provided $m \gtrsim s \log(d)$.

Sometimes, one knows more about x_0 than that it is sparse. Concretely, one might know that certain sets of indices $S_i \subseteq \{1, \ldots, d\}$ are more likely to contain non-zero indices. A proposed method for taking advantage of this additional knowledge is to use a *weighted* ℓ_1 -norm in the minimization program:

$$\min \|x\|_{1,w} = \min \sum_{i=1}^{d} w_i |x(i)| \text{ subject to } Ax = b.$$

Choosing the weights is a subtle issue. The general idea is to put lower weights on indices which are likely to be non-zero, but choosing the weights in a clever manner can lead to significantly better results. In this talk, we will describe a simple and efficient method of finding weights which in a certain sense are optimal for the case of A being Gaussian. The talk is based on the paper [1].

Références

 A. Flinth. Choice of Weights for Sparse Recovery With Prior Information. *IEEE Trans. Inf. Theory.* 62(7):4276 - 4284, 2016.