Stochastic control and optimal stopping game problems with nonlinear expectations

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In this lecture we review some recent developments of the theory of optimal stopping and stochastic control problems under nonlinear expectation. These nonlinear expectations are defined via nonlinear backward stochastic differential equations (BSDEs). We give main properties of these operators, and show how they can characterize dynamic risk measures and can be used in mathematical finance to define nonlinear pricing systems, also called g-conditional expectations [5, 7].
In this framework, we review some ideas and recent results for stochastic control, optimal stopping, Dynkin stochastic games under nonlinear expectation. In the Markovian case, nonlinear BSDEs provide general Feyman Kac probabilistic representations of nonlinear parabolic equations, and general dynamic programming principles can be established for controlled problems. Some illustrative examples dealing with the pricing and hedging of European, American and game options in financial markets with imperfections and with model ambiguity are given.

Références

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