Existence and uniqueness results for the pressureless Euler-Poisson system

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We study the Euler-Poisson system describing the evolution of a fluid without pressure effect and, more generally, a class of nonlinear hyperbolic systems with the same structure: More generally, in this paper we cover a class of systems:

$$\partial_t \rho + \partial_x(\rho f'(u)) = 0, \tag{1a}$$

$$\partial_t(\rho u) + \partial_x(\rho f'(u)u) = \rho h(\textstyle\int_{-\infty}^{x} \rho dy), \tag{1b}$$

with $u : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ and $\rho : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$. In the above, $h : \mathbb{R} \to \mathbb{R}$ is a given Lipschitz continuous function and $f : \mathbb{R} \to \mathbb{R}$ satisfies the following two assumptions:

1. $f \in C^2(\mathbb{R})$ is a strictly convex function;
2. $\lim\limits_{|x|\to\infty} \frac{f(x)}{|x|} = \infty$.

We investigate the initial value problem and generalize a method introduced by LeFloch in 1990 and based on Volpert's product and Lax's explicit formula. A well-posed theory is obtained when one component of the system is a measure-valued solution, while the second one has bounded variation. Existence is established for general initial data, while uniqueness is guaranteed only when the initial data does not generate rarefaction centers. We first solve a nonconservative version of the problem and construct solutions with bounded variation. The solutions to the systems of interest is then obtained by differentiation, which provides us with a complete theory of existence and uniqueness for both formulations.

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