From discrete microscopic models to macroscopic models and applications to traffic flow

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Mots-clés : specified homogenization, Hamilton-Jacobi equations, integro-differential operators, Slepčev formulation, viscosity solutions, traffic flow, microscopic models, macroscopic models.

In this work, we focus on deriving a macroscopic model for traffic flow problems from a microscopic model. The idea is to rescale the microscopic model, which describes the dynamics of each vehicle individually, in order to get a macroscopic model which describes the dynamics of density of vehicles.
The originality of our work is that we assume that there is a local perturbation that slows down the vehicles. At the microscopic scale, we consider a first order model of the form "follow the leader" i.e.

$$\dot{U}_j(t) = V\left(U_{j+1}(t) - U_j(t)\right) \cdot \phi\left(U_j(t)\right), \tag{1}$$

where U_j denotes the position of the jth vehicle and $\dot{U}_j$ is its velocity. The function $\phi : \mathbb{R} \to [0,1]$ simulates the presence of the local perturbation around the origin, for instance it can simulate the presence of a speed bump. The function V is called the optimal velocity function.
At the macroscopic scale, we obtain an explicit Hamilton-Jacobi equation left and right of the origin with a junction condition at the origin and a flux limiter. As it turns out, the macroscopic model is equivalent to a LWR model, with a flux limiting condition at the junction. Recently, concerning this type of equations, in [**?**], Imbert and Monneau give a suitable definition of (viscosity) solutions at the junction which allows to prove comparison principle, stability and so on.
The techniques used in this work use the ideas developed in [**?**] in order to pass from microscopic models to macroscopic ones. In particular, we will show that this problem can be seen as an homogenization result. The difficulty here is that, due to the local perturbation, we are not in a periodic setting and so the construction of suitable correctors is more complicated. In particular, we will use the idea developped in [**?**], [**?**] and in the lectures of Lions at the "College de France" [**?**], which consists in constructing correctors on truncated domains.
Finally, we also present qualitative properties concerning the flux limiter at the junction.

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