Finite element resolution of the stationnary Gross-Pitaevskii equation with rotation

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To simulate a rotating Bose-Einstein condensate we solve the stationary Gross-Pitaevskii equation:

$$-\frac{1}{2}\Delta u + V_{trap}u + C_g|u|^2u - iC_\Omega(y\partial_x u - x\partial_y u) = 0, \quad u \in H_0^1(\mathcal{D}), \quad \mathcal{D} \subset \mathbb{R}^3,$$

under the the unitary norm constraint $\int_{\mathcal{D}} |u|^2 = 1$. C_g and C_Ω are constants depending on the number of atoms and the rotation speed. We consider different trapping potentials, with the general expression (quartic + quadratic):

$$V_{trap} = \frac{1}{2}(a_x x^2 + a_y y^2 + a_z z^2 + a_4 r^4).$$

A projected Sobolev Gradient method adapted to this problem was derived in [?] and implemented with FreeFem++. To improve the efficiency of the method, a mesh adaptivity algorithm was developed in [?]. An alternative method using the Ipopt library [?] was suggested and tested by F. Hecht. In this work, we present some algorithmic and technical improvements of the two methods and their extensions for 3D cases. The developed programs are now available into a FreeFem++ Toolbox with a simple interface for both 2D and 3D configurations. After presenting the numerical methods, focus will be given to the description of the architecture and the use of the Toolbox. Finally, some 2D and 3D examples will illustrate this work.

References

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