Equivalent source modelling of small heterogeneities in the context of $3D$ time-domain wave propagation equation

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Short abstract *In the context of time harmonic wave equation, we are interested in the computation of the scattered field by a small obstacle. The result of a high performance direct numerical simulation is compared to an approximate solution derived by the method of matched asymptotic expansions.*

Introduction In the context of acoustic imaging, it is rather difficult to observe heterogeneities with characteristic length smaller than the wave length emitted by the scanner. However, it is possible to detect small heterogeneities in homogeneous media by using high performance computing. In this work, we will propose a way to compute the field scattered by a small obstacle with low computation burden based on the matched asymptotic expansions.

The considered problem Let us consider a small obstacle B_ε equipped with Dirichlet boundary conditions. The propagation domain Ω_ε consists of the exterior to the obstacle B_ε. We consider the solution of the $3D$ time-domain wave equation equipped with the Dirichlet boundary condition and homogeneous initial conditions.

Matching of asymptotic expansions The matching of asymptotic expansions method [1] is an asymptotic domain decomposition method with overlapping. It consists in representing the solution with a far-field expansion far away from the obstacle and a near-field expansion near the obstacle. These two expansions are matched in a transition zone with the so-called Van Dyke matching conditions. This approach is equivalent [3] to the corrector method [2].

Results Then, we present the results of a numerical experiment. We compare the second order far-field expansion to a direct numerical approximation of the exact solution achieved with an Interior Penalty Discontinuous Galerkin Method (IPDG) associated to a second order BGT absorbing boundary condition and to a local space-time mesh refinement [4]. Numerically, we observe that the relative error is lower than 6%.

Références

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