Asynchronous OpenCL/MPI Discontinous Galerkin Solver for Conservation laws[1]

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We consider the following general form of hyperbolic conservation laws for an unknown vector $W(x,t) \in \mathbb{R}^m$ that depends on a space variable $x = (x^1 \dots x^d)$, $d = 2$ or 3, and time t according to

$$\partial_t W + \sum_{k=1}^{k=d} \partial_k F^k(W) = S, \tag{1}$$

with the usual notations $\partial_t = \partial/\partial_t$ and $\partial_k = \partial/\partial x^k$. In equation (1), functions $F^k(W)$ and S characterize the physical model and we classically define the *flux* of the system by $F(W,n) = F^k(W)n_k$ for space vector $n = (n_1 \dots n_d) \in \mathbb{R}^d$. This very general mathematical framework can be applied to electromagnetism, fluid mechanics, multiphase flows, magneto-hydro-dynamics, Vlasov plasmas, etc.

We consider a mesh of the computational domain Ω. In each mesh cell L, we approximate the field by a linear combination of basis functions ψ_j^L,

$$W(x,t) = W_L^j(t)\psi_j^L(x), \quad x \in L, \tag{2}$$

and apply the Discontinuous Galerkin (DG) approximation,

$$\forall L, \forall i \quad \int_L \partial_t W \psi_i^L - \int_L F(W, W, \nabla\psi_i^L) + \int_{\partial L} F(W_L, W_R, n_{LR})\psi_i^L = 0, \tag{3}$$

where R is the neighbour cells along ∂L, n_{LR} the unit normal vector on ∂L oriented from L to R, and $F(W_L, W_R, n)$ is the numerical flux, which satisfies $F(W,W,n) = F^k(W)n_k$. The DG methods is well-suited for parallel computing on `GPU` (Graphic Processing Units): it requires a large amount of uniform and simple computations, relies on explicit time-integration, and can be formulated to present a regular and local data access pattern.

In this work, we explain how we have implemented a general DG method for multi-`GPU` computations with the `OpenCL` and `MPI` libraries in order to achieve high efficiency. We also explain how the basis is essentially built on Gauss-Legendre or Gauss-Lobatto points by tensor products of one-dimensional Lagrange polynomials. We thus have nice interpolation property and obtain a nodal basis [2].

Our choice of basis functions allows for several standard optimizations such as local memory prefetching, exploitation of the sparse nature of the tensor basis [1], etc. The implementation relies on a splitting of the DG mesh into sub-domains and sub-zones. We rely on the `OpenCL` asynchronous task graph in order to overlap `OpenCL` computations, memory transfers and `MPI` communications.

Références

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[2] S. Hesthaven and T. Warburton, *Nodal discontinuous Galerkin methods: algorithms, analysis, and applications*, volume 54. Springer Science & Business Media, 2007.

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