Compton scattering: anisotropic kinetic models and angular moments methods

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The Compton scattering describes the change of direction and energy of a photon interacting with an electron. From a mathematical point of view, this interaction is modelled by a non linear Boltzmann equation for photons [EMV]. For some applications, this equation is too costly to be discretized directly, since it involves integrals over pre and post collision momentum for the two particles. Under some physical assumptions, this Boltzmann equation can be approximated by the Kompaneets equation [KOM], which is a non linear Fokker-Planck type equation. An original method for the derivation of this equation from the Boltzmann equation has been proposed by Escobedo and co-authors in [EMV]. The main weakness of their derivation is that they assumed that the distribution function of the photons is isotropic, which may be problematic for our applications.

In this work we address the derivation of a hierarchy of anisotropic Kompaneets type equation describing the Compton scattering, using the same method but without assuming any isotropy for the distribution function. It leads to original models, which we couple to an equation describing the evolution of the electrons' macroscopic temperature. After proving some theoretical results for this coupled system, such as an H-theorem (entropy dissipation), we study some angular moments models [DF, BRU] (P_1 and M_1). Some numerical tests are performed in the context of these angular models to illustrate the influence of the Compton scattering in comparison with the case of classical isotropic scattering.

Références

[EMV] M. Escobedo, S. Mischler, M. A. Valle, *Homogeneous Boltzmann equation in quantum relativistic kinetic theory*, Electronic Journal of Differential Equations (04), 2003.

[KOM] A. S. Kompaneets *The establishment of thermal equilibrium between quanta and electrons*, Soviet Physics JETP 4 (5), 1957.

[BRU] T. A. Brunner *Forms of approximate radiation transport*, Sandia Report, 2002.

[DF] B. Dubroca, J. L. Feugeas *Theoretical and numerical study on a moment closure hierarchy for the radiative transfer equation*, CR. Acad. Sci. PAris 329 (1), pp 915-920, 1999.

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