LARGE DEVIATIONS OF THE REALIZED (CO-)VOLATILITY VECTOR

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Realized statistics based on high frequency returns have become very popular in financial economics. In recent years, different non-parametric estimators of the variation of a log-price process have appeared. These were developed by many authors and were motivated by the existence of complete records of price data. Among them are the realized quadratic (co-)variation which is perhaps the most well known example, providing a consistent estimator of the integrated (co-)volatility when the logarithmic price process is continuous. Limit results such as the weak law of large numbers or the central limit theorem have been proved in different contexts. In this paper, we propose to study the large deviation properties of realized (co-)volatility (i.e., when the number of high frequency observations in a fixed time interval increases to infinity. More specifically, we consider a bivariate model with synchronous observation schemes and correlated Brownian motions of the following form: $dX_{\ell,t} = \sigma_{\ell,t}dB_{\ell,t}+b_\ell(t,\omega)dt$ for $\ell = 1, 2$, where X_ℓ denotes the log-price, we are concerned with the large deviation estimation of the vector $V_t^n(X) = \big(Q_{1,t}^n(X), Q_{2,t}^n(X), C_t^n(X)\big)$ where $Q_{\ell,t}^n(X)$ and $C_t^n(X)$ represente the estimator of the quadratic variational processes $Q_{\ell,t} = \int_0^t \sigma_{\ell,s}^2 ds$ and the integrated covariance $C_t = \int_0^t \sigma_{1,s}\sigma_{2,s}\rho_s ds$ respectively, with $\rho_t = cov(B_{1,t}, B_{2,t})$. Our main motivation is to improve upon the existing limit theorems. Our large deviations results can be used to evaluate and approximate tail probabilities of realized (co-)volatility. As an application we provide the large deviation for the standard dependence measures between the two assets returns such as the realized regression coefficients up to time t, or the realized correlation. Our study should contribute to the recent trend of research on the (co-)variance estimation problems, which are quite often discussed in high-frequency financial data analysis.

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