Asymptotic of Sparse Support Recovery for Positive Measures

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We study sparse spikes deconvolution over the space of Radon measures when the input measure is a finite sum of positive Dirac masses using the BLASSO convex program. We focus on the recovery properties of the support and the amplitudes of the initial measure in the presence of noise when the minimum separation t of the input measure (the minimum distance between two spikes) tends to zero. We show that when $||w||_2/\lambda$, $||w||_2/t^{2n-1}$ and λ/t^{2n-1} are small enough (where λ is the regularization parameter, w the noise and n the number of spikes), which corresponds roughly to a sufficient signal-to-noise ratio, a noise level and a regularization parameter small enough with respect to the minimum separation, there exists a unique solution to the BLASSO program with exactly the same number of spikes as the original measure. We provide a first order expansion of this solution with respect to λ, the noise w and t, and we give explicit values of the constants involved. As a by-product of this first-order expansion, we show that the amplitudes and positions of the spikes of the solution both converge toward those of the input measure when the noise and the regularization parameter drops to zero faster than t^{2n-1}.

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