Compressed sensing with structured sparsity and structured acquisition

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Compressed Sensing (CS) is an appealing framework for applications such as Magnetic Resonance Imaging (MRI). However, the sensing schemes suggested by CS theories are made of random isolated measurements, which can be incompatible with the physics of acquisition. To reflect the physical constraints of the imaging device, we introduce the notion of blocks of measurements: the sensing scheme is not a set of isolated measurements anymore, but a set of groups of measurements which may represent any arbitrary shape (radial lines for instance). Structured acquisition with blocks of measurements are easy to implement, and they give good reconstruction results in practice [**?**]. However, very few results exist on the theoretical guarantees of CS reconstructions in this setting. In this work, we fill the gap between CS theory and acquisitions made in practice. To this end, the key feature to consider is the structured sparsity of the signal to reconstruct. The main contributions of this work are the following : (i) we provide recovery guarantees for vectors $x \in \mathbb{C}^n$ with a fixed support $S \subset \{1, \dots, n\}$. This is in strong contrast with the usual works that consider the reconstruction of arbitrary s-sparse vectors; (ii) we provide a theoretical justification to the use of block acquisitions in compressed sensing. By doing so, we enrich the family of sensing matrices available for compressed sensing.
The proposed theory has a few important consequences :

- the concepts of RIP or coherence are not sound anymore. They are replaced by new quantities which explicitely depend on the support S and the sensing vectors.
- the proposed theory allows envisionning the use of CS in situations that were not possible before. The use of incoherent transforms is not necessary anymore, given that the support S has some good properties.
- We show that a block structured acquisition can be used, only if the support structure is adapted to it. The resulting structures are more complex than the sparsity by levels of [**?**].
- The explicit dependency on the support S allows to provide guarantees of reconstruction for random signals with known distribution.

Références

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