A reduced basis finite element heterogeneous multiscale method for Stokes flow in porous media

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In this talk we present a numerical multiscale method for Stokes flow in porous media. We consider a two-scale model that is based on the Stokes homogenization results [4, 5, 6]. The presented method is named the *reduced basis Darcy-Stokes finite element heterogeneous multiscale method* (RB-DS-FE-HMM) [3], which is based on the DS-FE-HMM [1].
The (RB)-DS-FE-HMM methods use finite elements to solve a Darcy equation on a macroscopic mesh, where the missing permeability data are extracted from the Stokes micro problems at each macroscopic quadrature point. The computational cost of the DS-FE-HMM is heavily dominated by solving a large amount of micro problems. The RB-DS-FE-HMM is addressing this bottleneck by applying the reduced basis (RB) methodology [2] to the micro problems. We map the Stokes micro problems to a reference domain, where a single (fine) microscopic mesh is considered and the coefficients of the mapped problems are parametrized by macroscopic coordinate.
The RB framework is divided into two stages: an offline stage and an online stage. In the offline stage (performed only once) we use a greedy algorithm to choose a small set of parameters at which we numerically compute the Stokes micro problems (the reduced basis). Micro problems can be then projected into the low-dimensional solution space spanned by the reduced basis, which yields a (small but dense) reduced system. The online stage can be performed repeatedly and it provides a fast assembling and solution of the reduced system and a subsequent evaluation of the output of interest (effective permeability) for any parameter value (macroscopic coordinate). It can be shown that the presented method satisfies approximation and algebraic stability properties, while keeping a favorable computational cost that is independent of the microscopic mesh size.
A priori and a posteriori estimates of the RB-DS-FE-HMM can be derived and a residual-based adaptivity approach can be applied. Two and three dimensional numerical experiments confirm the accuracy of the RB-DS-FE-HMM and demonstrate the cost reduction compared to the DS-FE-HMM.

Références

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